KD EDUCATION ACADEMY [9582701166]

Time : 6 Hour | STD 11 Science Physics | Total Marks : 370

kd 90+ ch-13 Oscillations

* Choose The Right Answer From The Given Options.[1 Marks Each] [42]

1. The motion of satellites and planets is:
   (A) Periodic.
   (B) Oscillatory.
   (C) Simple harmonic.
   (D) Non-periodic.

   Ans. :
   a. Periodic.

   Explanation:

   The motion of planets and satellites are repetitive and repeats itself after a fixed interval of time. These type of motions are known as periodic motion.

2. A particle executing simple harmonic motion along y-axis has its motion described by the equation $y = A\sin(\omega t) + B$ The amplitude of the simple harmonic motion is:
   (A) A
   (B) B
   (C) A + B
   (D) $\sqrt{A+B}$

   Ans. :
   a. A

3. Two simple pendulums of length 5m and 10m respectively are given small linear displacement in one direction at the same time. They will be again in the phase when the pendulum of shorter length has completed oscillations:
   (A) 1
   (B) 2
   (C) 3
   (D) 5

   Ans. :
   b. 2

   Explanation:

   In case of simple pendulum,

   $T = 2\pi\sqrt{\frac{5}{g}}$ and $T' = 2\pi\sqrt{\frac{10}{g}}$

   $\therefore T' = 2T$

   Let the two pendulums are in same phase for first time when shorter one has completed n oscillations. Then

   $nT = (n-1)T'$

   $\frac{n}{n-1} = \frac{T'}{T} = \frac{2T'}{T} = 2$

   $n = 2n - 2$

   $n = 2$

4. The expression for displacement of an object in SHM is $x = A\cos(\omega t)$ The potential energy at $t = \frac{T}{4}$ is
   (A) $\frac{1}{2}kA^2$
   (B) $\frac{1}{8}kA^2$
   (C) $\frac{1}{4}kA^2$
   (D) Zero

   Ans. :

d. Zero

5. The equation of motion of a particle is $x = a\cos(at)^2$. The motion is:

(A) Periodic but not oscillatory. (B) Periodic and oscillatory.

(C) Oscillatory but not periodic. (D) Neither periodic nor oscillatory.

Ans. :

c. Oscillatory but not periodic.

Explanation:

Since x varies between -a and +a, the motion is oscillatory.

But as $\cos(\alpha t^2) = \cos(\alpha^2 t^2)$ does not have a period, the motion is not periodic.

6. Masses m and 3m are attached to the two ends of a spring of constant k. If the system vibrates freely, the period of oscillation will be:

(A) $\pi\sqrt{\frac{m}{k}}$ (B) $2\pi\sqrt{\frac{3m}{2k}}$

(C) $\pi\sqrt{\frac{3m}{k}}$ (D) $2\pi\sqrt{\frac{4m}{3k}}$

Ans. :

c. $\pi\sqrt{\frac{3m}{k}}$

7. The displacement of a particle is represented by the equation $y = \sin^3 \omega t$. The motion is:

(A) Non-periodic.

(B) Periodic but not simple harmonic.

(C) Simple harmonic with period $\frac{2\pi}{\omega}$.

(D) Simple harmonic with period $\frac{\pi}{\omega}$.

Ans. :

c. Simple harmonic with period $\frac{2\pi}{\omega}$.

Explanation:

All sine and cosine functions of t are simple harmonic in nature.

Hence the motion is simple harmonic motion.

A simple harmonic motion is always periodic.

$\text{Time period} = T' = \frac{2\pi}{\omega'}$

hence the motion is simple harmonic with time period $\frac{2\pi}{\omega}$.

8. Two simple harmonic motions of angular frequency 100rad/s^{-1} and 1000rad/s^{-1} have the same displacement amplitude. The ratio of their maximum acceleration is:

(A) 1 : 10 (B) 1 : 10 (C) 1 : 10 (D) 1 : 10

Ans. :

b. 1 : 10

9. A particle executing S.H.M. has a maximum speed of 30cm/ s and a maximum acceleration of 60cm/ s^2.The period of oscillation is:

(A) πs. (B) $\frac{\pi}{2}$ s. (C) 2π s. (D) $\frac{\pi}{t}$ s.

Ans. :

a. πs.

Explanation:

Key concept: Let equation of an SHM is represented by $y = a \sin \omega t$

$v = \frac{dy}{dt} = a\omega \cos \omega t$

Hence $(v)_{max} = a\omega$

Acceleration $(A) = \frac{dx^2}{dt^2} = -a\omega^2 \sin \omega t$

Hence $A_{max} = \omega^2 a$

Maximum speed, $v_{max} = \omega A$

Maximum acceleration, $a_{max} = \omega^2 A$

Divide eqn. (ii) by eqn. (i), we get

$\frac{a_{max}}{v_{max}} = \frac{\omega^2 A}{\omega A} = \omega$

$\therefore \frac{a_{max}}{v_{max}} = \frac{2\pi}{T}$

Here, $v_{max} = 30cms^{-1}, a_{max} = 60cms^2$

$\therefore T = 2\pi\left(\frac{30cms^{-1}}{60cms^{-2}}\right) = \pi s$

10. The displacement of a particle varies with time according to the relation:
$y = a \sin \omega t + b \cos \omega t$

(A) The motion is oscillatory but not S.H.M.

(B) The motion is S.H.M. with amplitude a + b.

(C) The motion is S.H.M. with amplitude $a^2 + b^2$

(D) The motion is S.H.M. with amplitude $\sqrt{a^2 + b^2}$.

Ans. :

d. The motion is S.H.M. with amplitude $\sqrt{a^2 + b^2}$.

Explanation:

key concept: The sum of two S.H.Ms of same frequencies is a S.H.M.

According to the question, the displacement

$y = a \sin \omega t + b \cos \omega t$

Given $x = a \sin \omega t + b \cos \omega t$

Let $a = A \cos \phi$

And $b = A \sin \phi$

Squaring and adding (ii) and (iii), we get

$a^2 + b^2 = A^2 \cos^2 \phi + A^2 \sin^2 \phi = A^2$

$= A^2 \Rightarrow A = \sqrt{a^2 + b^2}$

$y = A \sin \phi . \sin \omega t + A \cos \phi . \cos \omega t$

$= A \sin(\omega t + \phi)$

$\frac{dy}{dt} = A\omega \cos(\omega t + \phi)$

$\frac{d^2y}{dt^2} = -A\omega^2 \sin(\omega t + \phi)$

$= -Ay\omega^2 = (-A\omega^2)y$

$\Rightarrow \frac{d^2y}{dt^2} \propto (-y)$

Hence, it is an equation of SHM with amplitude

$A = \sqrt{a^2 + b^2}$

11. A particle is executing simple harmonic motion with frequency f. The frequency at which its kinetic energy changes into potential energy is:

(A) $\frac{f}{2}$ (B) f (C) 2f (D) 4f

Ans. :

c. 2f

Explanation:

Frequency of kinetic energy or potential energy is double than that of particle executing S.H.M.

12. Two particles P and Q describe SHM of same amplitude a and frequency v along the same straight line. The maximum distance between two particles is $\sqrt{2}$ a. The phase difference between the particles is:

(A) Zero (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{3}$

Ans. :

a. Zero

13. In SHM:

(A) PE is stored due to elasticity of system.

(B) KE is stored due to inertia of system.

(C) Both KE and PE are stored by virtue of elasticity of system.

(D) Both (a) and (b).

Ans. :

d. Both (a) and (b).

Exlanation:

In SHM, potential energy depends on its elastic behaviour and kinetic energy on its inertial behavior. In case of mass m oscillating on spring. KE is due to motion of m and PE is due to stretching of spring.

14. The periodic function $f(t) = A\sin(\omega t)$ repeats itself with periodic function of:

(A) 2π (B) 3π (C) π (D) $\frac{\pi}{2}$

Ans. :

a. 2π

Explanation:

A periodic function repeats itself after a time period T. and f(t) = f(t + T) As $\sin(\omega t + 2\omega\pi)$

$\therefore$ Period of function is

15. The equation of motion of a particle is $x = a\cos(\alpha t)^2$. The motion is:

(A) Periodic but not oscillatory. (B) Periodic and oscillatory.

(C) Oscillatory but not periodic. (D) Neither periodic nor oscillatory.

Ans. :

a. Oscillatory but not periodic.

Explanation:

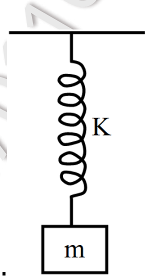
$x = a\cos(\propto t)^2$ is a cosine function and x varies between -a and +a, the motion is oscillatory. Now checking for periodic motion, putting t + T in place of t. T is supposed as period of the function ω(t).

$x(t+T) = a\cos[\alpha(t+T)]^2$

$= a\cos[\alpha^2 t^2 + a^2 T^2 + 2\alpha^2 tT] \neq x(t)$

Hence, it is not periodic.

16. A block is left in the equilibrium position as shown in the figure.



If now it is stretched by $\frac{mg}{k}$, the net stretch of the spring is:

(A) $\frac{mg}{k}$ (B) $\frac{mg}{2k}$ (C) $\frac{2mg}{k}$ (D) $\frac{mg}{4k}$

Ans. :

c. $\frac{2mg}{k}$

17. The ratio of frequencies of two pendulums oscillating are 2 : 3, then their lengths are in ratio:

(A) $\sqrt{\frac{2}{3}}$ (B) $\sqrt{\frac{3}{2}}$ (C) $\frac{4}{9}$ (D) $\frac{9}{4}$

Ans. :

d. $\frac{9}{4}$

18. The displacement of a particle is represented by the equation: $y = 3\cos\left(\frac{\pi}{4} - 2\omega t\right)$ The motion of the particle is:

(A) Simple harmonic with period $2\frac{P}{w}$.

(B) Simple harmonic with period $\frac{\pi}{\omega}$.

(C) Periodic but not simple harmonic.

(D) Non-periodic.

Ans. :

b. Simple harmonic with period $\frac{\pi}{\omega}$.

Explanation:

A simple harmonic motion is produced when a force (called restoring force) proportional to the displacement acts on a particle.

All sine and cosine functions of t are simple harmonic in nature.

Hence the motion is simple harmonic motion.

A simple harmonic motion is always periodic.

the motion is simple harmonic with time period $\frac{\pi}{\omega}$.

19. For a SHM, if the maximum potential energy become double, choose the correct option.

(A) Maximum kinetic energy will become double.

(B) The total mechanical energy will become double.

(C) Both (a) and (b) 39wisd.

(D) Neither (a) nor (b).

Ans. :

c. Both (a) and (b) 39wisd.

20. The rotation of earth about its axis is:

(A) Periodic motion.

(B) Simple harmonic motion.

(C) Periodic but not simple harmonic motion.

(D) Non-periodic motion.

Ans. :

a. Periodic motion.

c. Periodic but not simple harmonic motion.

Explanation:

Rotation of earth about its axis repeats its motion after a fixed interval of lime, so its motion is periodic.

The rotation of earth is obviously not a to and fro type of motion about a fixed point, hence its motion is not an oscillation. Also this motion does not follow S.H.M equation, $\mathrm{a} \propto -\mathrm{x}$

Hence, this motion is not a S.H.M.

21. The displacement of a particle in SHM varies according to the relation $\mathrm{x} = 4(\cos \pi \mathrm{t} + \sin \pi \mathrm{t})$ The amplitude of the particle is:

(A) -4

(B) 4

(C) $4\sqrt{2}$

(D) 8

Ans. :

c. $4\sqrt{2}$

Exlanation:

Given equation $\mathrm{x}(\mathrm{t}) = 4(\cos \pi \mathrm{t} + \sin \pi \mathrm{t})$

Now comparing cbove equation with general form $\mathrm{x}(\mathrm{t}) = \mathrm{A} \cos \omega \mathrm{t} + \mathrm{B} \sin \omega \mathrm{t}$

We get A = 4 and B = 4

As, the resultant amplitude for such a equation is

$= \sqrt{\mathrm{A}^2 + \mathrm{B}^2}$

$\therefore$ Amplitude $= \sqrt{4^2 + 4^2} = 4\sqrt{2}$

22. A body of mass 400g connected to a spring with spring constant $10 Nm^{-1}$, executes simple harmonic motion, time period of oscillation is

(A) $4\pi \times 10^{-1}\mathrm{s}$

(B) $0.3\pi \mathrm{s}$

(C) $2\mathrm{s}$

(D) $5 \times 10^{-1}\mathrm{s}$

Ans. :

a. $4\pi \times 10^{-1}\mathrm{s}$

Exlapation:

Here,m = 400g

400 × 10^{-3}kg

As R = 10Nm,$^{-1}$

$\mathrm{T} = 2\pi\sqrt{\frac{\mathrm{m}}{\mathrm{R}}}$

$= 2\pi\sqrt{\frac{400\times10^{-3}}{10}}$

$= 4\pi \times 10^{-1}\mathrm{s}$

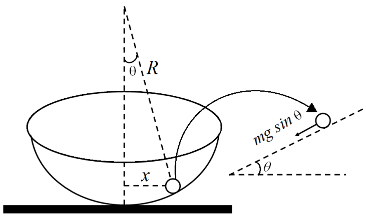
23. Motion of a ball bearing inside a smooth curved bowl, when released from a point slightly above the lower point is:

(A) Simple harmonic motion.
(B) Non-periodic motion.
(C) Periodic motion.
(D) Periodic but not S.H.M.

Ans. :

a. Simple harmonic motion.
c. Periodic motion.

Explanation:



For small angular displacement, the situation is shown in the figure. Only one restoring force creates.

motion in ball inside bowl.

Only one restoring force creates motion in ball inside bowl.

$\mathrm{F} = -\mathrm{mg}\sin\theta$

As θ is small, $\sin\theta = \theta$

So, $\mathrm{ma} = -\mathrm{mg}\frac{\mathrm{x}}{\mathrm{R}}$

Or, $\mathrm{a} = -\left(\frac{\mathrm{g}}{\mathrm{R}}\right)\mathrm{x} \Rightarrow \mathrm{a} \propto -\mathrm{x}$

So, motion of the ball is S.H.M and periodic.

24. At extreme position, velocity of the particle executing SHM that has amplitude A is:

(A) $\omega^2\mathrm{A}$
(B) 0
(C) $\omega\mathrm{A}$
(D) $\frac{\omega\mathrm{A}}{2}$

Ans. :

b. 0

Exlanation:

Velocity of the particle executing SHM is given as $\mathrm{v} = \omega\sqrt{\mathrm{A}^2 - \mathrm{x}^2}$

At extreme position, x = A

⇒ v = 0

25. A particle of mass m is executing oscillation about the origin about the origin on the x-axis, its potential energy is $U = kx^3$, where k is a positive constant. If the amplitude of oscillation is a, then its time period T is:

(A) Proportional to $\frac{1}{\sqrt{a}}$

(B) Independent of a

(C) Proportional to $\sqrt{a}$

(D) Proportional to $a^{\frac{3}{2}}$

Ans. :

a. Proportional to $\frac{1}{\sqrt{a}}$

Explanation:

Potential energy, $U = kx^3$

Force, $F = \frac{-dU}{dx} = -3kx^2$

max. force $= F_{max} - 3kx^2 = -m\omega^2 a$

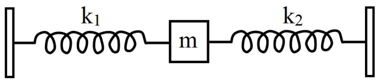
$\omega^2 = \frac{3ka^2}{ma} = \frac{3ka}{m}$

$\frac{4\pi^2}{T^2} = \frac{3ka}{m}$

$T^2 \propto \frac{1}{a}$

$T \propto \frac{1}{\sqrt{a}}$

26. Two spring of force constants k_1 and k_2 are connected to a mass m as shown in figure. The frequency of oscillation of the mass is f. If both k_1 and k_2 are made four times their original values, the frequency of oscillation becomes



(A) $\frac{f}{2}$

(B) $\frac{f}{4}$

(C) 4f

(D) 2f

Ans. :

d. 2f

Exlanation:

Frequency of oscillation,

$f = \frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}}$

$f' = \frac{1}{2\pi}.2\sqrt{\frac{k_1+k_2}{m}} = 2f$

27. A particle doing simple harmonic motion, amplitude = 4cm, time period = 12sec. Ratio of time taken by it in going from its mean position to 2cm and from 2cm to extreme position is:

(A) 1

(B) $\frac{1}{3}$

(C) $\frac{1}{4}$

(D) $\frac{1}{2}$

Ans. :

d. $\frac{1}{2}$

Explanation:

Here, a = 4cm; T = 12s. If t is the time takeo by particle in going fiom mean position to 2cm, then using

$\mathrm{y} = \mathrm{a}\sin\omega\mathrm{t}$, we have $2 = 4\sin\frac{2\pi}{\mathrm{T}}\mathrm{t}$

$\sin\frac{2\pi}{\mathrm{T}}\mathrm{t} = \frac{2}{4} = \frac{1}{2} = \sin\frac{\pi}{6}$

$\mathrm{t} = 1\mathrm{sec}.$

Time taken by particle to go from mean position to extreme position

$= \frac{\mathrm{T}}{4} = \frac{12}{4} = 3\mathrm{s}$

Therefore, time taken by Particle in going from 2cm to 4cm (i.e., extreme position)

$\mathrm{t}' = 3 - 1 = 2\mathrm{s}$

$\therefore$ So $\frac{\mathrm{t}}{\mathrm{t}'} = \frac{1}{2}$

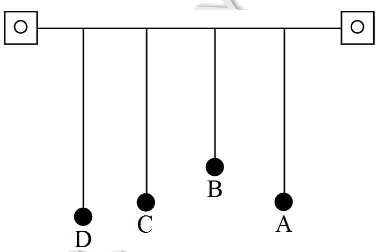
28. A particle executing SHM has a maximum speed of 30cm/ s angular frequency 10rad/ s. The amplitude of oscillation is:

(A) 3cm (B) 6cm (C) 1cm (D) 60cm

Ans. :

a. 3cm

29. Four pendulums A, B, C and D are suspended from the same elastic support as shown in Fig A and C are of the same length, while B is smaller than A and D is larger than A. If A



is given a transverse displacement,

(A) D will vibrate with maximum amplitude.

(B) C will vibrate with maximum amplitude.

(C) B will vibrate with maximum amplitude.

(D) All the four will oscillate with equal amplitude.

Ans. :

b. C will vibrate with maximum amplitude.

Explanation:

Here A is given a transverse displacement. Through the elastic support the disturbance is transferred to all the pendulums.

A and C are having same length, hence they will be in resonance, because of their time period of oscillation. Since length of pendulums A and C is same and $\mathrm{T} = 2\pi\sqrt{\frac{\mathrm{L}}{\mathrm{g}}}$ hence their time period is same and they will have frequency of vibration. Due to it, a resonance will take place and the pendulum C will vibrate with maximum amplitude.

30. The acceleration due to gravity on the surface of the moon is $1.7\mathrm{ms}^{-2}$. The time period of a simple pendulum on the moon, if its time period on the earth is 3.5s is:

(A) 2.2s (B) 4.4s (C) 8.4s (D) 16.8s

Ans. :

c. 8.4s

31. The motion of a swing is:

(A) Periodic but not oscillatory. (B) Oscillatory.

(C) Linear simple harmonic. (D) Circular motion.

Ans. :

b. Oscillatory.

32. A mass of 1kg attached to the bottom of a spring has a certain frequency of vibration. The following mass has to be added to it in order to reduce the frequency by half:

(A) 1kg (B) 2kg (C) 3kg (D) 4kg

Ans. :

c. 3kg

Explanation:

$\mathrm{v} = \frac{1}{2\pi}\sqrt{\frac{\mathrm{k}}{\mathrm{m}}}$ and $\frac{\mathrm{v}}{2} = \frac{1}{2\pi}\sqrt{\frac{\mathrm{k}}{\mathrm{m}_1}}$

$\therefore 2 = \sqrt{\frac{\mathrm{m}_1}{\mathrm{m}_2}}$

$\mathrm{m}_1 = 4\mathrm{m} = 4 \times 1 = 4\mathrm{kg}$

Hence mass added $= \mathrm{m}_1 - \mathrm{m} = 4 - 1 = 3\mathrm{kg}$

33. A mass M suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes S.H.M. of time period T. If the mass is increased by m, the time period becomes $\frac{5\mathrm{T}}{3}$. Then the ratio of $\frac{\mathrm{m}}{\mathrm{M}}$ is:

(A) $\frac{3}{5}$ (B) $\frac{25}{9}$ (C) $\frac{16}{9}$ (D) $\frac{5}{3}$

Ans. :

c. $\frac{16}{9}$

Explanation:

$\mathrm{T} = 2\pi\sqrt{\frac{\mathrm{M}}{\mathrm{k}}}$ and $\mathrm{T}' = 2\pi\sqrt{\frac{\mathrm{M}+\mathrm{m}}{\mathrm{k}}}$

$\frac{\mathrm{T}'}{\mathrm{T}} = \left(\frac{\mathrm{M}+\mathrm{m}}{\mathrm{M}}\right)^{\frac{1}{2}} = \left(1 + \frac{\mathrm{m}}{\mathrm{M}}\right)^{\frac{1}{2}}$

$\frac{\mathrm{m}}{\mathrm{M}} = \left(\frac{\mathrm{T}'}{\mathrm{T}}\right)^2 - 1 = \left(\frac{5}{3}\right)^2 - 1 = \frac{16}{9}$

34. The length of a simple pendulum is increased by 44%. What is the percentage increase in its time period?

(A) 10% (B) 20% (C) 40% (D) 44%

Ans. :

b. 20%

35. A horizontal platform with an object placed on it is executing S.H.M. in the vertical direction. The amplitude of oscillation is 3.92×10^{-3}m. At what time the object is not detached from the platform?

(A) 0.1256 sec. (B) 0.1356 sec. (C) 0.1456 sec. (D) 0.156 sec.

Ans. :

a. 0.1256 sec.

Explanation:

The object is not detached from the platform if

$mg = mr\omega^2 = mr\frac{4\pi r^2}{T^2}$

$T = 2\pi\sqrt{\frac{r}{g}} = 2 \times \frac{22}{7}\sqrt{\frac{3.29\times10^{-3}}{9.8}}$

$= 0.1256 \text{ sec}$

36. The displacement time graph of a particle executing S.H.M. is shown in Fig.Which of the following statement is/ are true?

a. The force is zero at $t = \frac{3T}{4}$.

b. The acceleration is maximum at $t = \frac{4T}{4}$.

c. The velocity is maximum at $t = \frac{T}{4}$.

d. The P.E. is equal to K.E. of oscillation at $t = \frac{T}{4}$.

Ans. :

a. The force is zero at $t = \frac{3T}{4}$.

b. The acceleration is maximum at $t = \frac{4T}{4}$.

c. The velocity is maximum at $t = \frac{T}{4}$.

Explanation:

a. At $t = \frac{3T}{4}$ Particle is at it's mean position so force acting on it is zero, but it continue the motion due to inertia of mass, here a = 0, so F = 0.

b. At $t = \frac{4T}{4} = T$,particle's velocity changes increasing to decreases so maximum change in velocity at T. As Acceleration$= \frac{\text{change in velocity}}{\text{Time}}$,so acceleration is maximum here.

c. $t = \frac{T}{2} = \frac{2T}{4}$,the particles has K.E = 0. So $KE \neq PE$.

37. The displacement of a particle is represented by the equation: $y = 3\cos\left(\frac{\pi}{4} - 2\omega t\right)$

The motion of the particle is:

a. Simple harmonic with period $2\frac{P}{w}$.

b. Simple harmonic with period $\frac{\pi}{\omega}$.

c. Periodic but not simple harmonic.

d. Non-periodic.

Ans. :

b. Simple harmonic with period $\frac{\pi}{\omega}$.

Explanation:

A simple harmonic motion is produced when a force (called restoring force) proportional to the displacement acts on a particle.

All sine and cosine functions of t are simple harmonic in nature.

Hence the motion is simple harmonic motion.

A simple harmonic motion is always periodic.

the motion is simple harmonic with time period $\frac{\pi}{\omega}$.

38. A wall clock uses a vertical spring-mass system to measure the time. Each time the mass reaches an extreme position, the clock advances by a second. The clock gives correct time at the equator. If the clock is taken to the poles it will:

a. Run slow.
b. Run fast.
c. Stop working.
d. Give correct time.

Ans. :

d. Give correct time.

Explanation:

Because the time period of a spring-mass system does not depend on the acceleration due to gravity.

39. The motion of a particle is given by $x = A\sin\omega t + B\cos\omega t$. The motion of the particle is:

a. Not simple harmonic.
b. Simple harmonic with amplitude A + B.
c. Simple harmonic with amplitude $\frac{(A+B)}{2}$.
d. Simple harmonic with amplitude $\sqrt{(A^2+B^2)}$.

Ans. :

d. Simple harmonic with amplitude $\sqrt{(A^2+B^2)}$.

Explanation:

$x = A\sin\omega t + B\cos\omega t \ldots (1)$

Acceleration,

$a = \frac{d^2x}{dt^2} = \frac{d^2}{dt^2}(A\sin\omega t + B\cos\omega t)$

$= \frac{d}{dt}(A\omega\cos\omega t - B\omega\sin\omega t)$

$= -A\omega^2\sin\omega t = B\omega^2\cos\omega t$

$= -\omega^2(A\sin\omega t + B\cos\omega t)$

$= -\omega^2 x$

For a body to undergo simple harmonic motion,

acceleration, a = -kx .....(2)

Therefore, from the equations (1) and (2), it can be seen that the given body undergoes simple harmonic motion with amplitude, $A = \sqrt{A^2+B^2}$.

40. The distance moved by a particle in simple harmonic motion in one time period is:

a. A
b. 2A
c. 4A
d. zero.

Ans. :

c. 4A

Explanation:

In an oscillation, the particle goes from one extreme position to other extreme position that lies on the other side of mean position and then returns back to the initial extreme position. Thus, total distance moved by particle is,

2A + 2A = 4A.

41. Two bodies A and B of equal mass are suspended from two separate massless springs of spring constant k_1 and k_2 respectively. If the bodies oscillate vertically such that their maximum velocities are equal, the ratio of the amplitude of A to that of B is:

a. $\frac{k_1}{k_2}$

b. $\sqrt{\frac{k_1}{k_2}}$

c. $\frac{k_2}{k_1}$

d. $\sqrt{\frac{k_2}{k^1}}$

Ans. :

d. $\sqrt{\frac{k_2}{k^1}}$

Explanation:

Maximum velocity, $v = A\omega$

where A is amplitude and ω is the angular frequency.

Further, $\omega = \sqrt{\frac{k}{m}}$

Let A and B be the amplitudes of particles A and B respectively. As the maximum velocity of particles are equal,

i.e. $v_A = v_B$

or,

$A\omega_A = B\omega_B$

$\Rightarrow A\sqrt{\frac{k_1}{m_A}} = B\sqrt{\frac{k_2}{m_B}}$

$\Rightarrow A\sqrt{\frac{k_1}{m}} = B\sqrt{\frac{k_2}{m}}\left(m_A = m_B = m\right)$

$\Rightarrow \frac{A}{B} = \sqrt{\frac{k_2}{k_1}}$

42. The total mechanical energy of a spring-mass system in 1 simple harmonic motion is $E = \frac{1}{2}m\omega^2A^2$. Suppose the oscillating particle is replaced by another particle of double the mass while the amplitude A remains the same. The new mechanical energy will:

a. Become 2E

b. Become $\frac{E}{2}$

c. Become $\sqrt{2E}$

d. Remain E.

Ans. :

d. Remain E.

Explanation:

Mechanical energy (E) of a spring-mass system in simple harmonic motion is given by,

$E = \frac{1}{2} m\omega^2 A^2$

where m is mass of body, and ω is angular frequency.

Let m_1 be the mass of the other particle and ω_1 be its angular frequency.

New angular frequency ω_1 is given by,

$\omega_1 = \sqrt{\frac{k}{m_1}} = \sqrt{\frac{k}{2m}} (m_1 = 2m)$

New energy E_1 is given as,

$E_1 = \frac{1}{2} m_1 \omega_1^2 A^2$

$= \frac{1}{2}(2m)\left(\sqrt{\frac{k}{2m}}\right)^2 A^2$

$= \frac{1}{2} m\omega^2 A^2 = E$

* Fill In The Blanks [2]

43. A child swinging on a swing in sitting position stands up then the time period of the swing will ________.

Ans. : A child swinging on a swing in sitting position stands up then the time period of the swing will **Decrease**.

44. The displacement y of a particle executing periodic motion is given by $y = 4\cos^2\left(\frac{t}{2}\right)\sin(1000t)$ This expression may be considered to be a result of the superposition of _______ independent harmonics.

Ans. : The displacement y of a particle executing periodic motion is given by

$y = 4\cos^2\left(\frac{t}{2}\right)\sin(1000t)$

This expression may be considered to be a result of the superposition of **Three** independent harmonics.

Explanation:

$y = 4\cos^2\left(\frac{t}{2}\right)\sin 1000t$

$= 2(1+\cos t)\sin 1000t \ (\because 2\cos^2\theta = 1 + \cos 2\theta)$

$= 2\sin 1000t + 2\sin 1000t \cos t$

$= 2\sin 1000t + \sin(1000+1)t \quad [\because 2\sin A\cos B = \sin(A+B) + \sin(A-B)]$

$+ \sin(1000-1)t$

$= 2\sin 1000t + \sin 1001t + \sin 999t$

This shows that the given expression is the result of three independent harmonics.

* Answer The Following Questions In One Sentence.[1 Marks Each] [2]

45. A platoon of soldiers marches on a road in steps according to the sound of a marching band. The band is stopped and the soldiers are ordered to break the steps while crossing a bridge. Why?

Ans. : Forced oscillation may break the bridge.

46. A small creature moves with constant speed in a vertical circle on a bright day. Does its shadow formed by the sun on a horizontal plane move in a simple harmonic motion?

Ans. : Yes, its shadow on a horizontal plane moves in simple harmonic motion. The projection of a uniform circular motion executes simple harmonic motion along its diameter (which is the shadow on the horizontal plane), with the mean position lying at the centre of the circle.

*** Given Section consists of questions of 3 marks each. [108]**

47. Plot the corresponding reference circle for each of the following simple harmonic motions. Indicate the initial (t = 0) position of the particle, the radius of the circle, and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anticlockwise in every case: (x is in cm and t is in s). $x = 3\sin\left(2\pi t + \frac{\pi}{4}\right)$

Ans. : $x = 3\sin\left(2\pi t + \frac{\pi}{4}\right)$

$$= -3\cos\left[\left(2\pi t + \frac{\pi}{4}\right) + \frac{\pi}{2}\right] = -3\cos\left(2\pi t + \frac{3\pi}{4}\right)$$

If this equation is compared with the standard SHM equation

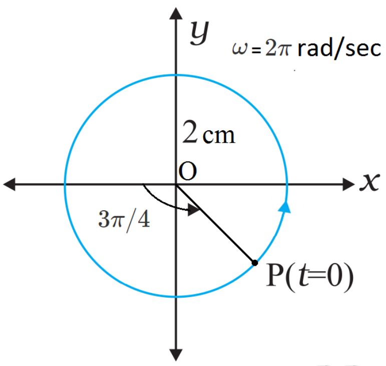
$x = A\cos\left(\left(\frac{2\pi}{T}\right)t + \phi\right)$, then we get:

Amplitude, A = 3cm

Phase angle, $\phi = \frac{3\pi}{4} = 135^\circ$

Angular velocity, $\omega = \frac{2\pi}{T} = 2\pi\ \mathrm{rad/s}$

The motion of the particle can be plotted as shown in the following figure.



48. A simple pendulum of length l and having a bob of mass M is suspended in a car. The car is moving on a circular track of radius R with a uniform speed v. If the pendulum makes small oscillations in a radial direction about its equilibrium position, what will be its time period?

Ans. : The bob of the simple pendulum will experience the acceleration due to gravity and the centripetal acceleration provided by the circular motion of the car.

Acceleration due to gravity = g

Centripetal acceleration = v^2/R

where,

v is the uniform speed of the car

R is the radius of the track

Effective acceleration (g') is given as:

$$g' = \sqrt{g^2 + \frac{v^4}{R^2}}$$

$\therefore$ Time period, $T = 2\pi\sqrt{\frac{l}{g'}}$

$$= 2\pi \frac{l}{g^2 + \frac{v^4}{R^2}}$$

49. A 0.2kg. of mass hangs at the end of a spring. When 0.02kg more mass is added to the end of the spring, it stretches 7cm more. If the 0.02kg mass is removed, what will be the period of vibration of the system?

Ans. : When 0.02kg is added, there is a stretch of 7cm. Using mg = Kx, we have

$$K = \frac{0.02 \times 10}{7 \times 10^{-2}} = \frac{20}{7} = 2.86 N/m$$

Time period $= T = 2\pi\sqrt{\frac{m}{K}}$

$$= 2\pi\sqrt{\frac{0.2}{2.86}} = 1.66 \text{ sec.}$$

50. For a particle in S.H.M., the displacement x of the particle as a function of time t is given as $x = A\sin(2\pi t)$. Here x is in cm and t is in seconds. Let the time taken by the particle to travel from x = 0 to $x = \frac{A}{2}$ be T_1 and the time taken to travel from $x = \frac{A}{2}$ to x = A be T_2. Find $\frac{T_1}{T_2}$.

Ans. : x = 0 at t = 0, t = 1s

Let $x = \frac{A}{2}$ at $t = T_1$

then $\frac{A}{2} = A\sin(2\pi T_1)$

$$A.\sin\left(\frac{\pi}{6}\right) = A\sin(2\pi T_1)$$

$$\Rightarrow \frac{\pi}{6} = 2\pi T_1$$

$$T_1 = \frac{1}{12} s$$

Time taken from x = 0 to x = A is $\frac{T}{4}$

$\Rightarrow$ at $x = A$ and $t = T$

$$A = A\sin 2\pi T$$

$$\sin\frac{\pi}{2} = \sin 2\pi T$$

$$T = \frac{1}{4} \text{sec}$$

$$T_1 + T_2 = \frac{1}{4}$$

$$T_2 = \frac{1}{4} - \frac{1}{12} = \frac{1}{6} s$$

So, $\frac{T_1}{T_2} = \frac{\frac{1}{12}}{\frac{1}{6}} = \frac{1}{2}$

51. What is a second's pendulum? How much is its length on the surface of moon?

Ans. : A second's pendulum is one whose time period of e. oscillation is 2 seconds. On the surface of moon,

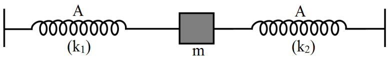
$a = \frac{g}{6}$.

$\therefore$ Using $T = 2\pi\sqrt{\frac{l}{g}}$,

We have $l = \frac{4g}{6 \times 4\pi^2} = \frac{1}{6} m$

52. A particle of mass 0.1kg is held between two rigid supports by two springs of force constants 8N/ m and 2N/ m. If the particle is displaced along the direction of the length of the springs, calculate its frequency of vibration.

Ans. : The situation is shown in the fig.



When the mass is displaced along the direction of the length of the spring, one spring is compressed while the other is extended but the force due to both the springs is in the same direction. Hence the effective force constant

$k = k_1 + k_2 = 8N/m + 2N/m = 10N/m$

The frequency of vibration is given by

$v = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$= \frac{1}{2\pi}\sqrt{\frac{10}{0.1}}$

$v = \frac{10}{2\pi}$

$= \frac{5}{\pi} s^{-1}$

53. A spring of force constant k has a mass M suspended from it. If the spring is cut into two halves, and the same mass is attached to one of the pieces, what will be the frequencies of oscillation of the mass?

Ans. : When the spring is cut into two equal halves, the force constant of each part will be doubled. Therefore, the original frequency, $v = \frac{1}{2\pi}\sqrt{\frac{k}{M}}$ will become

$v' = \frac{1}{2\pi}\sqrt{\frac{2k}{M}} = \sqrt{2v}$

54. The displacement of a particle executing periodic motion is given by:

$y = 4\cos^2\left(\frac{t}{2}\right)\sin(1000t)$. Find independent constituent simple harmonic motion.

Ans. : $y = 4\cos^2\left(\frac{t}{2}\right)\sin(1000t)$

$= 2(1 + \cos t)\sin(1000t)$ $[\because 2\cos^2\theta = 1 + \cos 2\theta]$

$= 2\sin 1000t + 2\sin 1000t \times \cos t$

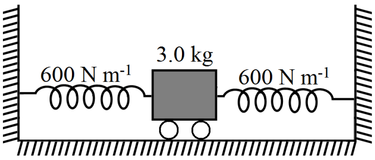
$= 2\sin 1000t + \sin(1000 + 1)t + \sin(1000 - 1)t$

$[\because 2\sin A\cos B = \sin(A + B) + \sin(A - B)]$

$= 2\sin 1000t + \sin 1001t + \sin 999t$

$= \sin 1000t + \sin 1000t + \sin 1001t + \sin 999t$

55. A trolley of mass 3.0kg, as shown in Figure, is connected to two springs, each of spring constant $600 Nm^{-1}$. If the trolley is displaced from its equilibrium position by 5.0cm and released, what is (a) the period of ensuing oscillations, and (b) the maximum speed of the trolley? How much energy is dissipated as heat by the time trolley comes to rest due damping forces?



Ans. : Equivalent spring constant: k' = 2k

$= 1200 Nm^{-1}$, m = 3kg

a. $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{3}{120}}$

$= \frac{2\pi}{20} = \frac{\pi}{10}\text{sec.}$

b. Maximum speed

$v = \omega A = 20 \times 5 \times 10^{-2} = 1ms^{-1}$

c. Energy dissipated = Maximum energy

$= \frac{1}{2}m\omega^2 A^2$

$= \frac{1}{2} \times 3 \times 20^2 \times 25 \times 10^{-4}$

$= 600 \times 25 \times 10^{-4}$

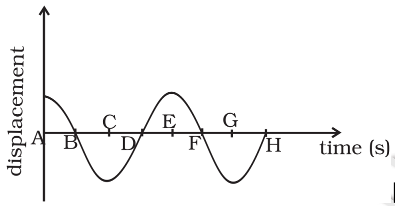
$= 150 \times 10^{-2}\text{Joule}$

$= 1.5 \text{ Jule}$

56. Displacement versus time curve for a particle executing S.H.M. is shown in Fig. Identify the points marked at which,

i. Velocity of the oscillator is zero,
ii. Speed of the oscillator is maximum.

Ans. :



Key concept: In displacement-time graph of SHM, zero displacement values correspond to mean position; where velocity of the oscillator is maximum. Whereas the crest and troughs represent amplitude positions, where displacement is maximum and velocity of the oscillator is zero.

i. The points A, C, E, G lie at extreme positions (maximum displacement, y = A). Hence the velocity of the oscillator is zero.
ii. The points B, D, F, H lie at mean position (zero displacement, y = 0). We know the speed is maximum at mean position.

57. A cylindrical wooden block of cross-section 15.0cm? and 230 grams is floated over water with an extra weight 50 grams attached to its bottom. The cylinder floats vertically. From the state of equilibrium, it is slightly depressed and released. If the specific gravity of wood is 0.3 and $g = 9.8ms^{-2}$, deduce the frequency of oscillation of the block.

Ans. : $mg = V\rho g$

$(230+50)\times 980 = 15.0\times l\times 1.0\times 980$

$l = \frac{280}{15\times 1.0} = 18.66cm$

$T = 2\pi\sqrt{\frac{18.66}{980}} = 0.8sec$

58. A body of mass 'm' suspended from a spring executes S.H.M. Calculate the ratio of the kinetic energy and potential energy of the body when it is at half the amplitude far from the mean position.

Ans. : $y = \frac{a}{2}$

$K.E. = \frac{1}{2}m\omega^2(a^2 - y^2)$

$P.E. = \frac{1}{2}m\omega^2 y^2$

$\frac{K.E.}{P.E.} = \frac{\frac{a^2 - a^2}{4}}{\frac{a^2}{4}} = \frac{3a^2}{4}\times\frac{4}{a^2}$

$\frac{K.E.}{P.E.} = \frac{3}{1}$

59. A force of 6.4N stretches a vertical spring by 0.1m. Find the mass that must be suspended from the spring so that it oscillates with the period of $\left(\frac{\pi}{4}\right)$ second.

Ans. : spring factor, $k = \frac{f}{m} = \frac{6.4}{0.1} = 64Nm^{-1}; T = \frac{\pi}{4}s;$

Inertial factor = mass suspended = m.

In S.H.M the time period is given by

$T = 2\pi\sqrt{\frac{\text{Inertial factor}}{\text{Spring factor}}}$

$\therefore \frac{\pi}{4} = 2\pi\sqrt{\frac{m}{64}}$ or $m = 1kg$

60. The angular velocity and amplitude of a simple pendulum is ω and r respectively. At a displacement x from the mean position, if its kinetic energy is T and potential energy is V, find the ratio of T to V.

Ans. : Kinetic energy at. x is

$T = \frac{1}{2}m\omega^2(A^2 - x^2)$

Potential energy at x is

$V = \frac{1}{2}m\omega^2 x^2$

$\frac{T}{V} = \frac{A^2 - x^2}{x^2} = \left(\frac{A^2}{x^2} - 1\right)$

61. A body oscillates with SHM according to the equation (in SI unit) $x = 5\cos\left[2\pi t + \frac{\pi}{4}\right]$
At t = 1.5 second, calculate (i) displecement, (ii) speed.

Ans. : $x = 5\cos\left[2\pi t + \frac{\pi}{4}\right]$ at $t = 1.5$ sec

Displacement, $x = 5\cos\left[2\pi(1.5) + \frac{\pi}{4}\right]$

$= 5\cos\left[3\pi + \frac{\pi}{4}\right]$

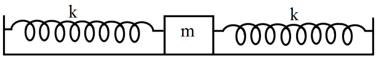
Velocity of oscillation,

$u = \frac{dx}{dt} = -5 \times 2\pi \times \sin\left(2\pi t + \frac{\pi}{4}\right)$

$= -10\pi \sin\left(2\pi t + \frac{\pi}{4}\right)$

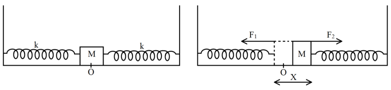
at $t = 1.5 \text{ sec}, u = -10\pi \sin\left(2\pi(1.5) + \frac{\pi}{4}\right)$

$= -10\pi \sin\left(3\pi + \frac{\pi}{4}\right)$

62. Two identical springs of spring constant k each are attached to a block of mass m as shown in figure: 

Show that when the mass is displaced from its equilibrium position on either side, it executes a simple harmonic motion. Find the period of oscillations.

Ans. :



Let the mass be displaced by a small distance x to the right of equilibrium position. Due to this, spring on left gets elongated by length equal to x and that on the right side gets compressed by same length. Then force acting on masses are

F_1 = -kx (force acting on left side and trying to pull the mass towards the mean position.)

F_2 = -kx (force exerted by spring on right side trying to push the mass towards mean position.)

Net force F, acting on the mass

F = -2kx

∴ Force acting on mass is directly propotional to displacement and it directed towards mean position.

∴ Motion is simple harmonic and time period of oscillation is

$T = 2\pi\sqrt{\frac{m}{2k}}$

63. What is the ratio of maxmimum acceleration to the maximum velocity of a simple harmonic oscillator?

Ans. : Consider a SHM. $x = A\sin\omega t$

$v = \frac{dx}{dt} = A\omega\cos\omega t$

For v_{max} $\cos\omega t = -1$

$\therefore v_{max} = A\omega$

$a = \frac{dv}{dt} = -A\omega^2\sin\omega t$

For a_{max} $\sin\omega t = -1$

$a_{max} = A\omega^2$

$\therefore \frac{a_{max}}{v_{max}} = \frac{A\omega^2}{A\omega} = \frac{\omega}{1}$

64.

i. What is meant by Simple Harmonic Motion (S.H.M)?

ii. At what points is the energy entirely kinetic and potential in S.H.M?

iii. What is the total distance travelled by a body executing S.H.M in a time equal to its time period, if its amplitude is A?

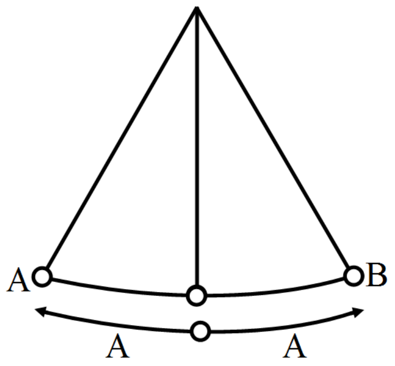
Ans. :

i. Simple harmonic motion is the projection of uniform circular motion on a diameter of a circle of reference.

ii. At mean position - K.E.

At extreme position - P.E.

iii. 4A because in completing one oscillation it crosses mean position 2 times so total dist. is 4A.



65. Two simple harmonic motions are represented by: $x_1 = 10\sin\left(4\pi t + \frac{\pi}{4}\right)$

$x_2 = 5(\sin 4\pi t + \sqrt{3}\cos 4\pi t)$ What is the ratio of the amplitudes?

Ans. : $x_2 = 5\sin 4\pi t + 5\sqrt{3}\cos 4\pi t$

Amplitude of $x_2 = \sqrt{5^2 + (5\sqrt{3})^2} = 10$

Since the $\sin \pi t$ and $\cos 4\pi t$ functions are out ofphase by $\frac{\pi}{2}$.

Amplitude of x_2 = 10

∴ Ratio of amplitudes is 1 : 1

66. A cylindrical piece of cork of density of base area A and height h floats in a liquid of density ρ_l. The cork is depressed slightly and then released. Show that the cork oscillates up and down simple harmonically with a period. $T = 2\pi\sqrt{\frac{h\rho}{\rho_l g}}$ where ρ is the density of cork. (Ignore damping due to viscosity of the liquid).

Ans. : Given, Area of cross-section of cork = A, Height of cork = h

Density of liquid $= \rho_l$

In equilibrium position of the weight of liquid displacement by cork = weight of cork. When cork is further depressed by ξ, than it further displaces liquid, an extra upthrust acts upwards, which provides restoring force to the cork.

Restoring force = extra upthrust

= weight of extra displaced water.

$F = -(\text{volume} \times \text{density} \times g)$

$F = -A \times y \times \rho_l \times g \ldots (i)$

$k = \frac{F}{y} = -A\rho_l g$

The period of oscillation of cork is given by

$\mathrm{T}=2\pi\sqrt{\frac{\mathrm{m}}{\mathrm{k}}}$

where m = mass of cork

= volume of cork × density of cork

$=\mathrm{A}\times\mathrm{h}\times\rho$

$\mathrm{T}=2\pi\sqrt{\frac{\mathrm{A}\times\mathrm{h}\times\rho}{\mathrm{A}\rho_{\mathrm{l}}\mathrm{g}}}=2\pi\sqrt{\frac{\mathrm{h}\rho}{\rho_{\mathrm{l}}\mathrm{g}}}$

thus, $\mathrm{T}=2\pi\sqrt{\frac{\mathrm{h}\rho}{\rho_{\mathrm{l}}\mathrm{g}}}$

67. A body oscillates with S.H.M. according to the equation: $\mathrm{x}(\mathrm{t})=5\cos\left(2\pi\mathrm{t}+\frac{\pi}{4}\right)$ where x is in meters and t is in seconds. Calculate the following:

a. Displacement at t = 0

b. Angular frequency

c. Magnitude of velocity (Maximum).

Ans. : $\mathrm{x}(\mathrm{t})=5\cos\left(2\pi\mathrm{t}+\frac{\pi}{4}\right)\mathrm{m}$

a. $\mathrm{x}(0)=5\cos\left(0+\frac{\pi}{4}\right)=5\times\frac{1}{\sqrt{2}}\mathrm{m}=\frac{5}{\sqrt{2}}\mathrm{m}$

b. Angular frequency $=\omega=2\pi\mathrm{v}\ \mathrm{rad/sec.}$

c. Maximum velocity $=\omega\mathrm{A}=2\pi\times5=10\pi\ \mathrm{ms}^{-1}$

68. A body of mass 5kg executes S.H.M. of amplitude of 0.5m. If the force constant is $100\mathrm{Nm}^{-1}$, calculate

i. Its time period.

ii. Its maximum kinetic energy, maximum potential energy and total energy.

Ans. : Given: m = 5kg, k = 100N/m; A = 0.5m

i. Time period is given by

$\mathrm{T}=2\pi\sqrt{\frac{\mathrm{m}}{\mathrm{k}}}=2\pi\sqrt{\frac{5}{100}}=0.41\mathrm{s}$

ii. Angular velocity,

$\omega=\frac{2\pi}{\mathrm{T}}=\frac{2\pi}{0.41}=4.5\ \mathrm{rad/\ s}$

Maximum $\mathrm{K.E.}=\mathrm{E}_{\mathrm{Kmax}}=\frac{1}{2}\mathrm{mv}_0^2$

$=\frac{1}{2}\mathrm{m}(\omega\mathrm{A})^2=12.50\mathrm{J}$

Maximum $\mathrm{P.E.}=\mathrm{E}_{\mathrm{Pmax}}$

$=\frac{1}{2}\mathrm{kA}^2=12.50\mathrm{J}$

Total energy $\mathrm{E}=\mathrm{E}_{\mathrm{Kmax}}$

$=\mathrm{E}_{\mathrm{Pmax}}=12.50\mathrm{J}$

69. A harmonic oscillation is represented by $\mathrm{y}=0.34\cos(3000\mathrm{t}+0.74)$, where y and t are in m and s respectively. Deduce: (i) the amplitude, (ii) the frequency and angular frequency, (iii) the period, and (iv) the initial phase.

Ans. : Given that, $\mathrm{y}=0.34\cos(3000\mathrm{t}+0.74)$

While the general expression for displacement is, $\mathrm{y}=\mathrm{a}\cos(\omega\mathrm{t}+\phi_0)$

Comparing these two expressions,

i. Amplitude, a = 0.34m.

ii. Angular frequency

$\omega = 3000 \text{ radian}/\sec^{-1}$

Frequency $v = \frac{\omega}{2\pi} = \frac{3000}{2}\pi = \frac{1500}{\pi}\text{Hz.}$

iii. Period $T = \frac{1}{v} = \frac{1}{\frac{1500}{\pi}} = \frac{\pi}{1500}\text{s.}$

iv. Initial phase $\phi_0 = 0.74 \text{ rad.}$

70. Plot the corresponding reference circle for each of the following simple harmonic motions. Indicate the initial (t = 0) position of the particle, the radius of the circle, and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anticlockwise in every case: (x is in cm and t is in s). $x = -2\sin\left(3t + \frac{\pi}{3}\right)$

Ans. : $x = -2\sin\left(3t + \frac{\pi}{3}\right) = +2\cos\left(3t + \frac{\pi}{3} + \frac{\pi}{2}\right)$

$= 2\cos\left(3t + \frac{5\pi}{6}\right)$

If this equation is compared with the standard SHM equation

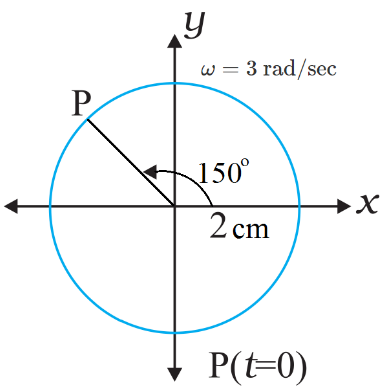
$x = A\cos\left(\left(\frac{2\pi}{T}\right)t + \phi\right)$, then we get:

Amplitude, A = 2cm

Phase angle, $\phi = \frac{5\pi}{6} = 150^\circ$

Angular velocity, $\omega = \frac{2\pi}{T} = 3\text{rad/sec.}$

The motion of the particle can be plotted as shown in the following figure.



71. The displacement of a particle having S.H.M. is x = 10 sin $\left[10\pi t + \frac{\pi}{4}\right]$m.

i. Amplitude.

ii. Angular frequency.

iii. Epoch.

iv. Time period.

v. Frequency.

vi. Maximum velocity.

Ans. : Given equation is, $x = 10\sin\left(10\pi t + \frac{\pi}{4}\right)\text{m}$, comparing with $x = A\sin(\omega t + \phi)$ we have

i. Amplitude, $A = 10\text{m.}$

ii. Angular frequency, $\omega = 10\pi$.

iii. Epoch = initial phase, $\frac{\pi}{4}$.

iv. Time period, $\mathrm{T} = \frac{1}{5}\mathrm{sec}$.

v. Frequency, $\mathrm{f} = 5\mathrm{Hz}$.

vi. Maximum velocity $\omega \mathrm{A} = 100\pi \mathrm{ms}^{-1}$.

72. Derive the expression for resultant spring constant when two springs having constants k_1 and k_2 are connected

i. In parallel.

ii. In series.

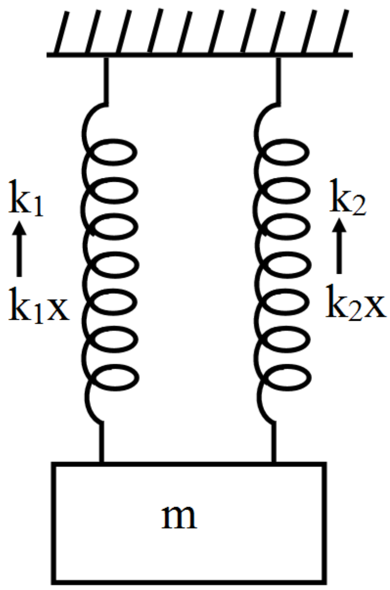
Ans. :

i. When the springs are connected in parallel, the extension in them will be same and the total restoring force is the sum of their restoring forces.

$\therefore \mathrm{F} = \mathrm{F}_1 + \mathrm{F}_2$

$-\mathrm{k}_{\mathrm{eq}}\mathrm{x} = -\mathrm{k}_1\mathrm{x} - \mathrm{k}_2\mathrm{x}$

$\mathrm{k}_{\mathrm{eq}} = \mathrm{k}_1 + \mathrm{k}_2$



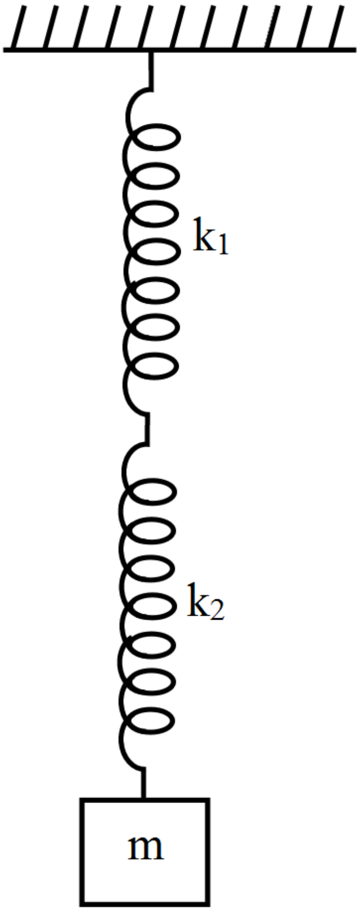
ii. When the springs are connected in series, the restoring force is same in both the springs and the extensions will be different so the not extension

i.e., $\mathrm{x} = \mathrm{x}_1 + \mathrm{x}_2$

$= \frac{\mathrm{F}}{-\mathrm{k}_{\mathrm{eq}}} = \frac{-\mathrm{F}}{\mathrm{k}_1} - \frac{\mathrm{F}}{\mathrm{k}_2}$

$\therefore \frac{1}{\mathrm{k}_{\mathrm{eq}}} = \frac{1}{\mathrm{k}_1} + \frac{1}{\mathrm{k}_2}$

when connected in series.



73. Find the displacement of a simple harmonic oscillator at which its PE is half of the maximum energy of the oscillator.

Ans. : Let us assume that the required displacement where PE is half of the maximum energy of the oscillator be x.

The potential energy of the oscillator at this position,

$\mathrm{PE} = \frac{1}{2}\mathrm{kx}^2 = \frac{1}{2}\mathrm{m}\omega^2\mathrm{x}^2$

maximum energy of the oscillator = maximum potential energy = Total energy

$\mathrm{TE} = \frac{1}{2}\mathrm{m}\omega^2\mathrm{A}^2$

Where, A = amplitude of motion.

We are given, $\mathrm{PE} = \frac{1}{2}\mathrm{TE}$

$\Rightarrow \frac{1}{2}\mathrm{m}\omega^2\mathrm{x}^2 = \frac{1}{2}\left[\frac{1}{2}\mathrm{m}\omega^2\mathrm{A}^2\right]$

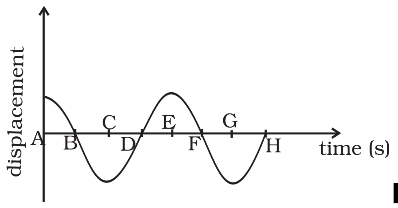
$\Rightarrow \mathrm{x}^2 = \frac{\mathrm{A}^2}{2} \text{ or } \mathrm{x} = \sqrt{\frac{\mathrm{A}^2}{2}} = \pm\frac{\mathrm{A}}{\sqrt{2}}$

74. Displacement versus time curve for a particle executing S.H.M. is shown in Fig. Identify the points marked at which,

i. Velocity of the oscillator is zero,

ii. Speed of the oscillator is maximum.

Ans. :



Key concept: In displacement-time graph of SHM, zero displacement values correspond to mean position; where velocity of the oscillator is maximum. Whereas the crest and troughs represent amplitude positions, where displacement is maximum and velocity of the oscillator is zero.

i. The points A, C, E, G lie at extreme positions (maximum displacement, y = A). Hence the velocity of the oscillator is zero.

ii. The points B, D, F, H lie at mean position (zero displacement, y = 0). We know the speed is maximum at mean position.

75. Show that the motion of a particle represented by $y = \sin ax - \cos cot$ is simple harmonic with a period of $\frac{2\pi}{\omega}$.

Ans. : A function will represent S.H.M. if it can be written uniquely in the form of a or a sin $\left(\frac{2\pi}{T}t + \phi\right)$

Now $y = \sin \omega t - \cos \omega t$

$y = \sqrt{2}\left[\sin \omega t \frac{1}{\sqrt{2}} - \cos \omega t \sin \frac{\pi}{4}\right]$

$y = \sqrt{2} \sin\left(\omega t - \frac{\pi}{4}\right)$

Comparing with standard SHM $y = a \sin\left(\frac{2\pi}{T}t + \phi\right)$

$w = \frac{2\pi}{T}$ or $T = \frac{2\pi}{\omega}$.

76. What is the ratio of maxmimum acceleration to the maximum velocity of a simple harmonic oscillator?

Ans. : Consider a SHM. $x = A \sin \omega t$

$v = \frac{dx}{dt} = A\omega \cos \omega t$

For v_{max} $\cos \omega t = -1$

$\therefore v_{max} = A\omega$

$a = \frac{dv}{dt} = -A\omega^2 \sin \omega t$

For a_{max} $\sin \omega t = -1$

$a_{max} = A\omega^2$

$\therefore \frac{a_{max}}{v_{max}} = \frac{A\omega^2}{A\omega} = \frac{\omega}{1}$

77. The pendulum of a certain clock has time period 2.04s. How fast or slow does the clock run during 24 hours?

Ans. : The pendulum of the clock has time period 2.04sec.

Now, No. or oscillation in 1 day $= \frac{24 \times 3600}{2} = 43200$

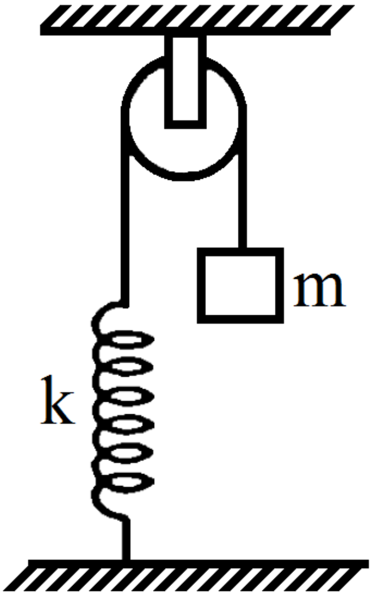
But, in each oscillation it is slower by $(2.04 - 2.00) = 0.04\,\text{sec}$.

So, in one day it is slower by,

= 43200 × (0.04) = 12sec = 28.8min

So, the clock runs 28.8 minutes slower in one day.

78. The string, the spring and the pulley shown in figure are light. Find the time period of



the mass m.

Ans. : When only 'm' is hanging, let the extension in the spring be $'\ell'$

So $T_1 = k\ell = mg.$

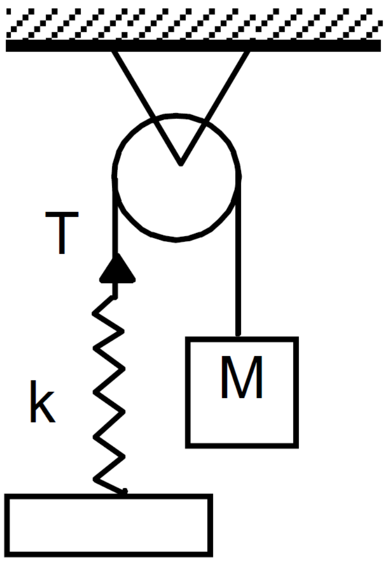
When a force F is applied, let the further extension be 'x'

$\therefore T_2 = k(x + \ell)$

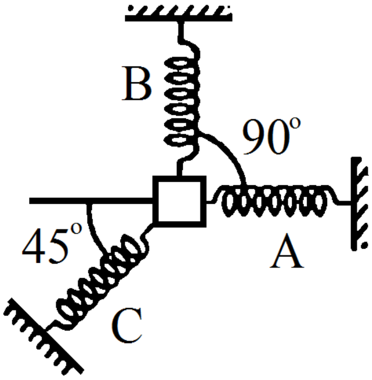
$\therefore$ Driving force $= T_2 - T_1 = k(x + \ell) - k\ell = kx$

$\therefore$ Acceleration $= \frac{k\ell}{m}$

$T = 2\pi\sqrt{\frac{\text{displacement}}{\text{Acceleration}}} = 2\pi\sqrt{\frac{x}{\frac{kx}{m}}} = 2\pi\sqrt{\frac{m}{k}}$



79. A particle of mass in is attatched to three springs A, B and C of equal force constants k as shown in figure If the particle is pushed slightly against the spring C and released,



find the time period of oscillation.

Ans. : Suppose the particle is pushed slightly against the spring 'C' through displacement 'x'.

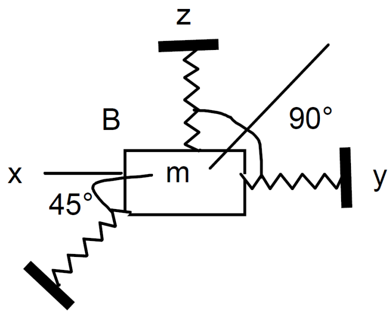
Total resultant force on the particle is kx due to spring C and $\frac{kx}{\sqrt{2}}$ due to spring A and B.

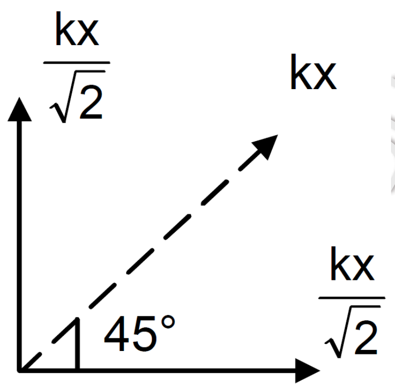
$\therefore$ Total Resultant force $= kx + \sqrt{\left(\frac{kx}{\sqrt{2}}\right)^2 + \left(\frac{kx}{\sqrt{2}}\right)^2} = kx + kx = 2kx.$

Acceleration $= \frac{2kx}{m}$

Time period $T = 2\pi\sqrt{\frac{\text{displacement}}{\text{Acceleration}}} = 2\pi\sqrt{\frac{x}{\frac{2kx}{m}}} = 2\pi\sqrt{\frac{m}{2k}}$

[**Cause:** When the body pushed against 'C' the spring C, tries to pull the block towards XL. At that moment the spring A and B tries to pull the block with force $\frac{kx}{\sqrt{2}}$ and $\frac{kx}{\sqrt{2}}$ respectively towards xy and xz respectively. So the total force on the block is due to the spring force 'C' as well as the component of two spring force A and B.]





80. A spring stores 5J of energy when stretched by 25cm. It is kept vertical with the lower end fixed. A block fastened to its other end is made to undergo small oscillations. If the block makes 5 oscillations each second, what is the mass of the block?

Ans. : $x = 25cm = 0.25m$

$E = 5J$

$f = 5$

So, $T = \frac{1}{5}$sec.

Now, $P.E = \left(\frac{1}{2}\right)kx^2$

$\Rightarrow \left(\frac{1}{2}\right)kx^2 = 5$

$\Rightarrow \left(\frac{1}{2}\right)k(0.25)^2 = 5$

$\Rightarrow k = 160N/m.$

Again, $T = 2\pi\sqrt{\frac{m}{k}}$

$\Rightarrow \frac{1}{5} = 2\pi\sqrt{\frac{m}{160}}$

$\Rightarrow m = 0.16kg.$

81. A block of mass 0.5kg hanging from a vertical spring executes simple harmonic motion of amplitude 0.1m and time period 0.314s. Find the maximum force exerted by. the spring on the block.

Ans. : $x = r = 0.1m$

$T = 0.314\,sec$

$m = 0.5kg.$

Total force exerted on the block = weight of the block + spring force.

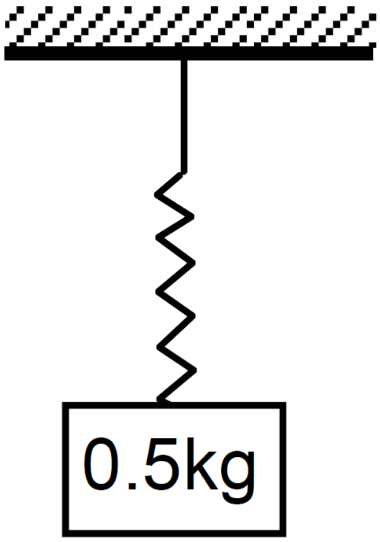
$T = 2\pi\sqrt{\frac{m}{k}}$

$\Rightarrow 0.314 = 2\pi\sqrt{\frac{0.5}{k}}$

$k = 200N/m$

$\therefore$ Force exerted by the spring on the block is $F = kx = 201.1 \times 0.1 = 20N$

$\therefore$ Maximum force $= F + Weight = 20 + 5 = 25N$



82. The maximum tension in the string of an oscillating pendulum is double of the minimum tension. Find the angular amplitude.

Ans. : The tension in the pendulum is maximum at the mean position and minimum on the extreme position.

Here $\left(\frac{1}{2}\right)\mathrm{mv}^2 - 0 = \mathrm{mg}\ell(1-\cos)\theta$

$\mathrm{v}^2 = 2\mathrm{g}\ell(1-\cos\theta)$

Now, $\mathrm{T_{max}} = \mathrm{mg} + 2\mathrm{mg}(1-\cos)\theta \left[\mathrm{T} = \mathrm{mg} + \left(\frac{\mathrm{mv}^2}{\ell}\right)\right]$

Again, $\mathrm{T_{min}} = \mathrm{mg}\cos\theta$

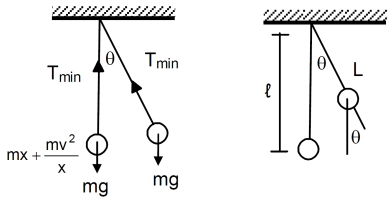
According to question, $\mathrm{T_{max}} = 2\mathrm{T_{min}}$

$\Rightarrow \mathrm{mg} + 2\mathrm{mg} - 2\mathrm{mg}\cos\theta = 2\mathrm{mg}\cos\theta$

$\Rightarrow 3\mathrm{mg} = 4\mathrm{mg}\cos\theta$

$\Rightarrow \cos\theta = \frac{3}{4}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{3}{4}\right)$



*** Given Section consists of questions of 5 marks each.** **[200]**

83. A block whose mass is $1kg$ is fastened to a spring. The spring has a spring constant of $50Nm^{-1}$. The block is pulled to a distance $x = 10cm$ from its equilibrium position at $x = 0$ on a frictionless surface from rest at $t = 0$. Calculate the kinetic, potential and total energies of the block when it is $5cm$ away from the mean position.

Ans. : The block executes SHM, its angular frequency, as given by Eq. (13.14b), is

$$\omega = \sqrt{\frac{k}{m}}$$

$$= \sqrt{\frac{50Nm^{-1}}{1kg}}$$

$$= 7.07rads^{-1}$$

Its displacement at any time t is then given by,

$x(t) = 0.1\cos(7.07t)$

Therefore, when the particle is $5cm$ away from the mean position, we have

$0.05 = 0.1\cos(7.07t)$

Or $\cos(7.07t) = 0.5$ and hence

$\sin(7.07t) = \frac{\sqrt{3}}{2} = 0.866$

Then, the velocity of the block at $x = 5cm$ is

$= 0.1 \times 7.07 \quad 0.866ms^{-1}$
$= 0.61ms^{-1}$

Hence the K.E. of the block,
$= \frac{1}{2}mv^2$
$= 1/2\left[1kg \times \left(0.6123ms^{-1}\right)^2\right]$
$= 0.19J$

The P.E. of the block,
$= \frac{1}{2}kx^2$
$= 1/2\left(50Nm^{-1} \times 0.05m \times 0.05m\right)$
$= 0.0625J$

The total energy of the block at $x = 5cm$,
$= \text{ K.E.} + \text{P.E.}$
$= 0.25J$
we also know that at maximum displacement, K.E. is zero and hence the total energy of the system is equal to the P.E. Therefore, the total energy of the system,
$= 1/2\left(50Nm^{-1} \times 0.1m \times 0.1m\right)$
$= 0.25J$
which is same as the sum of the two energies at a displacement of $5cm$. This is in conformity with the principle of conservation of energy.

84. You are riding in an automobile of mass 3000kg. Assuming that you are examining the oscillation characteristics of its suspension system. The suspension sags 15cm when the entire automobile is placed on it. Also, the amplitude of oscillation decreases by 50% during one complete oscillation. Estimate the values of (a) the spring constant k and (b) the damping constant b for the spring and shock absorber system of one wheel, assuming that each wheel supports 750kg.

Ans. :

a. Mass of the automobile, m = 3000kg
Displacement in the suspension system, x = 15cm = 0.15m
There are 4 springs in parallel to the support of the mass of the automobile.
The equation for the restoring force for the system:
F = –4kx = mg
Where, k is the spring constant of the suspension system
Time period, $\mathrm{T} = 2\pi\sqrt{\frac{\mathrm{m}}{4\mathrm{k}}}$
and k = mg/4x = 3000 × 10/4 × 0.15 = 5000 = 5 × 10^4Nm
Spring Constant, k = 5 × 10^4Nm

b. Each wheel supports a mass, M = 3000/4 = 750kg

For damping factor b, the equation for displacement is written as

$x = x_0e^{-bt/2M}$

The amplitude of oscilliation decreases by 50%.

$\therefore x = x_0/2$

$x_0/2 = x_0e^{-bt/2M}$

$\log_e 2 = bt/2M$

$\therefore b = 2M \log_e 2/t$

where,

Time period, $t = 2\pi\sqrt{\frac{m}{4k}} = 2\pi\sqrt{\frac{3000}{4\times5\times10^4}} = 0.7691 \text{ s}$

$\therefore\ b = \frac{2\times750\times0.693}{0.7691} = 1351.58 \text{ kg/s}$

85. A body describes simple harmonic motion with an amplitude of 5cm and a period of 0.2s. Find the acceleration and velocity of the body when the displacement is (a) 5cm (b) 3cm (c) 0cm.

Ans. : Amplitude, A = 5cm = 0.05m

Time period, T = 0.2s

For displacement, x = 5cm = 0.05m

Acceleration is given by:

$a = -\omega^2 x$

$= -\left(\frac{2\pi}{T}\right)^2 x$

$= -\left(\frac{2\pi}{0.2}\right)^2 \times 0.05$

$= -5\pi^2 \text{ m/s}^2$

Velocity is given by:

$v = \omega\sqrt{A^2 - x^2}$

$= \frac{2\pi}{T}\sqrt{(0.05)^2 - (0.05)^2}$

= 0

When the displacement of the body is 5cm, its acceleration is $-5\pi^2 \text{ m/s}^2$ and velocity is 0.

For displacement, x = 3cm = 0.03m

$a = -\omega^2 x$

$= -\left(\frac{2\pi}{T}\right)^2 x$

$= -\left(\frac{2\pi}{0.2}\right)^2 0.03$

$= -3\pi^2 \text{ m/s}^2$

Velocity is given by:

$v = \omega\sqrt{A^2 - x^2}$

$= \frac{2\pi}{T}\sqrt{A^2 - x^2}$

$= \frac{2\pi}{T}\sqrt{(0.05)^2 - (0.03)^2}$
$= \frac{2\pi}{0.2} \times 0.04$
$= 0.4\pi \text{ m/s}$

When the displacement of the body is 3cm, its acceleration is $-3\pi \text{ m/s}^2$ and velocity is $0.4\pi \text{ m/s}.$

For displacement, x = 0

Acceleration is given by:

$a = -\omega^2 x = 0$

Velocity is given by:

$v = \omega\sqrt{A^2 - x^2}$
$= \frac{2\pi}{T}\sqrt{A^2 - x^2}$
$= \frac{2\pi}{0.2}\sqrt{(0.05)^2 - 0}$
$= 0.5\pi \text{ m/s}$

When the displacement of the body is 0, its acceleration is 0 and velocity is $0.5\pi \text{ m/s}.$

86. A circular disc of mass 10kg is suspended by a wire attached to its centre. The wire is twisted by rotating the disc and released. The period of torsional oscillations is found to be 1.5s. The radius of the disc is 15cm. Determine the torsional spring constant of the wire. (Torsional spring constant α is defined by the relation $J = -\alpha\theta$, where J is the restoring couple and θ the angle of twist).

Ans. : Mass of the circular disc, m = 10kg

Radius of the disc, r = 15cm = 0.15m

The torsional oscillations of the disc has a time period, T = 1.5s

The moment of inertia of the disc is:

$l = \frac{1}{2}mr^2$
$= \frac{1}{2} \times (10) \times (0.15)^2$
= 0.1125kg m^2

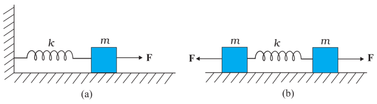
Time period, $T = 2\pi\sqrt{\frac{l}{\alpha}}$

α is the torsional constant.

$\alpha = \frac{4\pi^2 l}{T^2}$
$= \frac{4 \times (\pi)^2 \times 0.1125}{(1.5)^2}$
= 1.972Nm/rad

Hence, the torsional spring constant of the wire is 1.972Nm rad^{-1}.

87. Figure (a) shows a spring of force constant k clamped rigidly at one end and a mass m attached to its free end. A force F applied at the free end stretches the spring. Figure (b) shows the same spring with both ends free and attached to a mass m at either end. Each end of the spring in Fig. (b) is stretched by the same force F.



If the mass in Fig. (a) and the two masses in Fig. (b) are released, what is the period of oscillation in each case?

Ans. : In Fig. (a) if x is the extension in the spring, when mass m is returning to its mean position after being released free, then restoring force on the mass iss F = -kx, i.e., $\mathrm{F} \propto \mathrm{x}$
As, this F is directed towards mean position of the mass, hence the mass attached to the spring will execute SHM.

Spring factor = spring constant = k

inertia factor = mass of the given mass = m

As time period,

$$\mathrm{T} = 2\pi\sqrt{\frac{\text{inertia factor}}{\text{spring factor}}}$$

$$\therefore\ \mathrm{T} = 2\pi\sqrt{\frac{\mathrm{m}}{\mathrm{k}}}$$

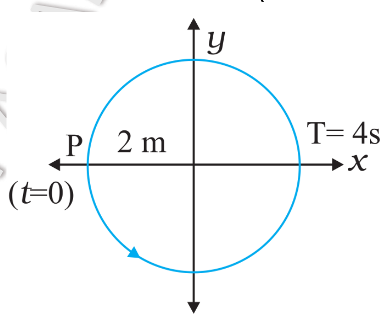
In Fig. (b), we have a two body system of spring constant k and reduced mass,
$\mu = \frac{\mathrm{m}\times\mathrm{m}}{\mathrm{m}+\mathrm{m}} = \frac{\mathrm{m}}{2}$

Inertia factor = m/2

Spring factor = k

$\therefore$ Time period, $\mathrm{T} = 2\pi\sqrt{\frac{\frac{\mathrm{m}}{2}}{\mathrm{k}}} = 2\pi\sqrt{\frac{\mathrm{m}}{2\mathrm{k}}}$

88. Figures correspond to two circular motions. The radius of the circle, the period of revolution, the initial position, and the sense of revolution (i.e. clockwise or anti-clockwise) are indicated on each figure.



Obtain the corresponding simple harmonic motions of the x-projection of the radius vector of the revolving particle P, in each case.

Ans. : Let B be any point on the circle of reference of the figure. From B draw BN perpendicular on x-axis.

Then $\triangle\mathrm{BON} = \theta = \omega\mathrm{t}$

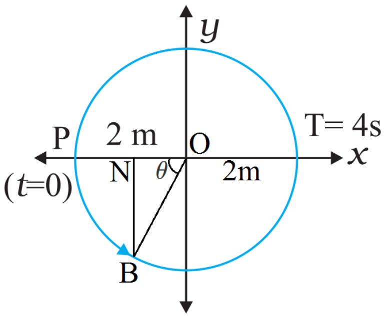
$\therefore$ In $\triangle\mathrm{ONB},\ \cos\theta = \frac{\mathrm{ON}}{\mathrm{OB}}$

Or $\mathrm{ON} = \mathrm{OB}\cos\theta$

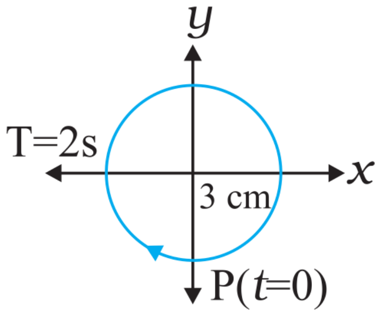
$$\therefore\ \ -\mathrm{x} = 2\cos\omega\mathrm{t}$$

$$\Rightarrow\ \mathrm{x} = -2\frac{\cos(2\pi)}{\mathrm{T}}\mathrm{t} = -2\cos\frac{2\pi}{4}\mathrm{t}$$

$\therefore\ \mathrm{x} = -2\cos\frac{\pi}{4}\mathrm{t}$ which is equation of SHM



89. Figures correspond to two circular motions. The radius of the circle, the period of revolution, the initial position, and the sense of revolution (i.e. clockwise or anti-clockwise) are indicated on each figure.



Obtain the corresponding simple harmonic motions of the x-projection of the radius vector of the revolving particle P, in each case.

Ans. : Let A be any point on the circle of reference of the figure From A, draw BN perpendicular on x-axis.

If $\angle \mathrm{POA} = \theta$, then

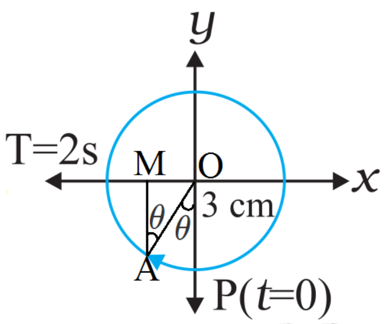
$\angle \mathrm{OAM} = \theta = \omega \mathrm{t}$

$\therefore$ In triangle OAM,

$\frac{\mathrm{OM}}{\mathrm{OA}} = \sin\theta$

$\therefore \frac{-\mathrm{x}}{3} = \omega \mathrm{t} = \frac{\sin(2\pi)}{\mathrm{T}}\mathrm{t}$

$\therefore \mathrm{x} = -3\frac{\sin(2\pi)}{2}\mathrm{t}$ or $\mathrm{x} = -3\sin\pi\mathrm{t}$ which is the equation of SHM.



90. A body of mass m is attached to one end of a massless spring which is suspended vertically from a fixed point. The mass is held in hand so that the spring is neither stretched nor compressed. Suddenly the support of the hand is removed. The lowest position attained by the mass during oscillation is 4cm below the point, where it was held in hand.

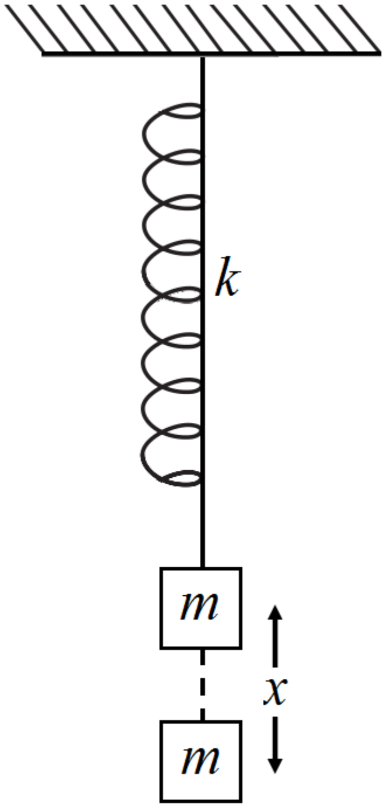
a. What is the amplitude of oscillation?

b. Find the frequency of oscillation?

Ans. : When mass m is held in support by hand the extension in spring will be zero as no deforming force acts on spring.

Let the mass reaches at its new position unit displacement from previous.

Then P.E. of spring or mass = gravitational P.E. lost by man.



P.E = mg x

But P.E. due to spring is $\frac{1}{2}\mathrm{kx}^2 \mathrm{k} = \omega^2 \mathrm{A}$

$\therefore \frac{1}{2}\mathrm{kx}^2 = \mathrm{mg\ x}$

Mean position of spring by block will be when let extension is then

$\mathrm{F} = -\mathrm{kx}_0$

$\mathrm{F} = \mathrm{mg} \therefore \mathrm{mg} = +\mathrm{kx}_0$ or $\mathrm{x}_0 = \frac{\mathrm{mg}}{\mathrm{k}} \ldots (\mathrm{ii})$

From (i) and (ii)

$\mathrm{x} = 2\left(\frac{\mathrm{mg}}{\mathrm{k}}\right) = 2\mathrm{x}_0$

$\mathrm{x} = 4\mathrm{cm} \therefore 4 = 2\mathrm{x}_0$

$\mathrm{x}_0 = 2\mathrm{cm}$

The amplitude of oscillator is the maximum distance from mean position.

$\mathrm{x} - \mathrm{x}_0 = 4 - 2 = 2\mathrm{cm}$

Time Period $\mathrm{T} = 2\pi\sqrt{\frac{\mathrm{m}}{\mathrm{k}}}$ which does not depend on amplitude.

$\frac{2\mathrm{mg}}{\mathrm{k}} = \mathrm{x}$ from (1)

$\frac{\mathrm{m}}{\mathrm{k}} = \frac{\mathrm{x}}{2\mathrm{g}} = \frac{4\times10^{-2}}{2\times9.8}$ or $\frac{\mathrm{k}}{\mathrm{m}} = \frac{2\times9.8}{4\times10^{-2}}$

$\mathrm{v} = \frac{1}{2\pi}\sqrt{\frac{\mathrm{k}}{\mathrm{m}}} = \frac{1}{2\times3.14}\sqrt{\frac{2\times9.8}{4\times10^{-2}}} = \frac{4.9\times10^{-2}}{6.28}$

$\mathrm{v} = \frac{10\times2.21}{6.28} = 3.52\mathrm{Hz}.$

Oscillator will not rise above the positive from where it was released because total extension in spring is 4cm when released and amplitude is 2cm.

So it oscillates below the released position.

91. A circular disc of mass 10kg is suspended by a wire attached to its centre. The wire is twisted by rotating the disc and released. The period of torsional oscillations is found to be 1.5s. The radius of the disc is 15cm. Determine the torsional spring constant of the wire. (Torsional spring constant α is defined by the relation $\mathrm{J} = -\alpha\theta$, where J is the restoring couple and θ the angle of twist).

Ans. : Mass of the circular disc, m = 10kg

Radius of the disc, r = 15cm = 0.15m

The torsional oscillations of the disc has a time period, T = 1.5s

The moment of inertia of the disc is:

$\mathrm{l} = \frac{1}{2}\mathrm{mr}^2$

$= \frac{1}{2} \times (10) \times (0.15)^2$

= 0.1125kg m^2

Time period, $\mathrm{T} = 2\pi\sqrt{\frac{\mathrm{l}}{\alpha}}$

α is the torsional constant.

$\alpha = \frac{4\pi^2\mathrm{l}}{\mathrm{T}^2}$

$= \frac{4\times(\pi)^2\times 0.1125}{(1.5)^2}$

= 1.972Nm/rad

Hence, the torsional spring constant of the wire is 1.972Nm rad^{-1}.

92. An 8kg body performs S.H.M. of amplitude 30cm. The restoring force is 60N when the displacement is 30cm. Find (a) time period (b) the acceleration, P.E. and K.E., when displacement is 12cm.

Ans. : Here, m = 8kg; a = 30cm = 0.30m;

$\mathrm{F} = 60\mathrm{N}; \mathrm{y} = 0.30\mathrm{m}$

$\mathrm{F} = -\mathrm{ky}$

$\mathrm{k} = -\frac{\mathrm{F}}{\mathrm{y}} = -\frac{60}{0.30} = -200\mathrm{Nm}^{-1}$

As $\omega = \sqrt{\frac{\mathrm{k}}{\mathrm{m}}} = \sqrt{\frac{200}{8}} = 5\mathrm{s}^{-1}$

Time period, $\mathrm{T} = \frac{2\pi}{\omega} = \frac{2\times 22}{7\times 5} = \frac{44}{35} = 1.256\mathrm{s}$

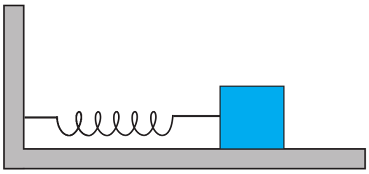
Here, $\mathrm{y} = 12\mathrm{cm} = 0.12\mathrm{m}$

$\therefore$ Acceleration, $\mathrm{A} = \omega^2\mathrm{y} = (5)^2 \times 0.12 = 3.0\mathrm{ms}^{-2}$

$\mathrm{P.E.} = \frac{1}{2}\mathrm{ky}^2 = 1.44\mathrm{J}$

$\mathrm{K.E.} = \frac{1}{2}\mathrm{k}(\mathrm{a}^2 - \mathrm{y}^2) = 7.56\mathrm{J}$

93. A spring having with a spring constant 1200N m^{-1} is mounted on a horizontal table as shown in Fig. A mass of 3kg is attached to the free end of the spring. The mass is then



pulled sideways to a distance of 2.0cm and released.
Determine (i) the frequency of oscillations, (ii) maximum acceleration of the mass, and (iii) the maximum speed of the mass.

Ans. : Spring constant, k = 1200N m^{-1}

Mass, m = 3kg

Displacement, A = 2.0cm = 0.02cm

i. Frequency of oscillation v, is given by the relation:

$$\upsilon = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

where, T is time period

$$\therefore\ \upsilon = \frac{1}{2\times 3.14}\sqrt{\frac{1200}{3}}$$

= 3.18m/s

Hence, the frequency of oscillations is 3.18 cycles per second.

ii. Maximum acceleration (a) is given by the relation:

$$a = \omega^2 A$$

where,

$\omega =$ Angular frequency $= \sqrt{\frac{k}{m}}$

A = maximum displacement

$$\therefore\ a = \frac{k}{m}A = \frac{1200\times 0.02}{3} = 8\ ms^{-2}$$

Hence, the maximum acceleration of the mass is 8.0m/s^2.

Maximum velocity, $v_{max} = A\omega$

$$= A\sqrt{\frac{k}{m}} = 0.02 \times \sqrt{\frac{1200}{3}} = 0.4\ m/s$$

Hence, the maximum velocity of the mass is 0.4m/s.

94. An 8kg body performs S.H.M. of amplitude 30cm. The restoring force is 60N when the displacement is 30cm. Find (a) time period (b) the acceleration, P.E. and K.E., when displacement is 12cm.

Ans. : Here, m = 8kg; a = 30cm = 0.30m;

F = 60N; y = 0.30m

F = -ky

$$k = -\frac{F}{y} = -\frac{60}{0.30} = -200Nm^{-1}$$

As $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{8}} = 5s^{-1}$

Time period, $T = \frac{2\pi}{\omega} = \frac{2\times 22}{7\times 5} = \frac{44}{35} = 1.256s$

Here, y = 12cm = 0.12m

$\therefore$ Acceleration

$$A = \omega^2 y = (5)^2 \times 0.12 = 3.0ms^{-2}$$

$\mathrm{P.E.} = \frac{1}{2}\mathrm{ky}^2 = 1.44\mathrm{J}$

$\mathrm{K.E.} = \frac{1}{2}\mathrm{k}(\mathrm{a}^2 - \mathrm{y}^2) = 7.56\mathrm{J}$

95. A simple pendulum with a brass bob has a time period T. The bob is now immersed in a non-viscous liquid and oscillated. If the density of the liquid is $\frac{1}{9}$ that of brass, find the time of the same pendulum.

Ans. : Let V be the volume and ρ be the density of the brass bob. Mass of the bob $\mathrm{m} = \mathrm{V}\rho$ and weight of bob $= \mathrm{V}\rho\mathrm{g}$. Buoyancy force of liquid on bob $= \mathrm{V}\left(\frac{\rho}{9}\right)\mathrm{g} = \frac{\mathrm{V}\rho\mathrm{g}}{9}$.

So, the effective weight of bob in liquid $= \mathrm{V}\rho\mathrm{g} - \frac{\mathrm{V}\rho\mathrm{g}}{9} = \frac{8\mathrm{V}\rho\mathrm{g}}{9}$,

$\therefore \text{Acceleration } \mathrm{g}' = \frac{\frac{8\mathrm{V}\rho\mathrm{g}}{9}}{\mathrm{m}}$

$= \frac{\frac{8\mathrm{V}\rho\mathrm{g}}{9}}{\mathrm{V}\rho} = \frac{8\mathrm{g}}{9}$

Time period of the bob

$-2\pi\sqrt{\frac{\mathrm{l}}{\mathrm{g}}} = 2\pi\sqrt{\frac{\mathrm{l}}{\left(\frac{8\mathrm{g}}{9}\right)}}$

$= 2\pi\sqrt{\frac{\mathrm{l}}{\mathrm{g}}} \times \frac{3}{\sqrt{8}} = \frac{3\mathrm{T}}{\sqrt{8}} \left(\because \mathrm{T} = 2\pi\sqrt{\frac{\mathrm{l}}{\mathrm{g}}}\right)$

96. The displacement x (in cm) of an oscillating particle varies with time t (in seconds) according to the equation. $\mathrm{x} = 2\cos(0.5\pi\mathrm{t} + \frac{\pi}{3})$ Find

a. Amplitude of oscillation.

b. The time period of oscillation.

c. The maximum velocity of the particle.

d. The maximum acceleration of the particle.

Ans. : The displacement of the particle is given by $\mathrm{x} = 2\cos(0.5\pi\mathrm{t} + \frac{\pi}{3})\mathrm{cm}$ To find the amplitude and time period of the oscillation, we compare this equation with

$\mathrm{x} = \mathrm{A}\cos(\omega\mathrm{t} + \delta)$

a. Amplitude A = 2cm

b. Angular frequency $\omega = 0.5\pi \text{ rad s}^{-1}$

$\mathrm{T} = \frac{2\pi}{\omega} = \frac{2\pi}{0.5\pi} = 4\mathrm{s}$

c. Maximum acceleration $\mathrm{a}_{\max} = |\mathrm{A}\omega|$

$= 2 \times 0.5\pi = \pi\mathrm{cms}^{-1} = 3.142\mathrm{cms}^{-1}$

d. Maximum acceleration $\mathrm{a}_{\max} = |-\omega^2\mathrm{A}|$

$= \omega^2\mathrm{A} = (0.5\pi)^2 \times 2$

$= \frac{\pi}{2}\mathrm{cms}^{-2} = 4.935\mathrm{cms}^{-2}$

97. A particle of mass m is executing simple harmonic oscillations of amplitude A. At $\mathrm{x} = \frac{\mathrm{A}}{2}$ what fraction of its energy is potential? What fraction is kinetic?

Ans. : We know that total energy of a harmonic oscillator

$\mathrm{E} = \frac{1}{2}\mathrm{m}\omega^2\mathrm{A}^2$

At $\mathrm{x} = \frac{\mathrm{A}}{2}$, the potential energy of oscillator,

$U = \frac{1}{2}m\omega^2x^2$

$\frac{1}{2}m\omega^2.\left(\frac{A}{2}\right)^2$

$= \frac{1}{8}m\omega^2A^2$

$\therefore \frac{U}{E} = \frac{\frac{1}{8}m\omega^2A^2}{\frac{1}{2}m\omega^2A^2}$

$= \frac{1}{4} = \frac{1}{4} \times 100\%$

$= 25\%$

$x = \frac{A}{2}$ the Kinetic energy

$K = \frac{1}{2}m\omega^2(A^2 - x^2)$

$= \frac{1}{2}m\omega^2\left[A^2 - \left(\frac{A}{2}\right)^2\right]$

$= \frac{3}{8}m\omega^2A^2$

$\therefore \frac{K}{E} = \frac{\frac{3}{8}m\omega^2A^2}{\frac{1}{2}m\omega^2A^2}$

$= \frac{3}{4} = \frac{3}{4} \times 100\%$

$= 75\%$

98. A body of mass 12kg is suspended by coil spring of natural length 50cm and force constant 2.0×10^3 Nm^{-1}. What is the stretched length of the spring? If the body is pulled down further strething the spring to a length of 5.9cm and then released, then what is the frequency of oscillation of the suspended mass? (Neglect the mass of the spring)

Ans. : Given m = 12kg,

Original length l = 50cm,

k = 20×10^3 Nm^{-1}

As, F = ky

$\therefore y = \frac{F}{k}$

$= \frac{mg}{k}$

$= \frac{12\times9.8}{2\times10^3}$

$= 5.9 \times 10^{-2}m$

$= 5.9cm$

$\therefore$ Stretched length of the spring $= l + y$

$= 50 + 55.9cm$

$= 105.9cm$

Frequency of oscillations, $V = \frac{1}{T}$

$= \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$= \frac{1}{2\times3.14}\sqrt{\frac{2\times10^3}{12}}$

$= 2.06s^{-1}$

99. The displacement of two particles executing simple harmonic motion are represented by equations, $y = 4\sin(10t + \theta)$ and $y = 5\cos 10t$ What is the phase difference between the velocities of these particles?

Ans. : For 1st particle

$y_1 = 4\sin(10t + \theta);$

Velocity $\frac{dy_1}{dt}$

$= 4 \times 10\cos(10t + \theta)$

$= 40\cos(10t + \theta)$

For second particle

$y_2 = 5\cos 10t = 5\sin(10t + \frac{\pi}{2})$

Velocity $\frac{dy_2}{dt}$

$= 5\cos \times 10\cos(10t + \frac{\pi}{2})$

$= 5\cos(10t + \frac{\pi}{2})$

Phase difference between velocities

$= (10t + \theta) - (10t + \frac{\pi}{2})$

$= \left(\theta - \frac{\pi}{2}\right)$

100. The number of harmonic components in the oscillations are represented by, $y = 4\cos^2$ at $\sin 4t$. What are What are their corresponding angular frequencies?

Ans. : $y = 4\cos^2 2t\sin 4t$

$= 2(\cos 4t + 1)\sin 4t$

$[\because 2\cos^2\theta = \cos 2\theta + 1]$

$= 2\sin 4t\cos 4t + 2\sin 4t$

$= \sin 8t + 2\sin 4t$

$= 2\sin 4t + \sin 8t$

The resulting harmonic oscillation is a combination of two harmonic motions of angular frequencies 4rad/ s and 8rad/ s.

101. A particle of mass 0.8kg is executing simple harmonic motion with amplitude of 1.0 metre and periodic time $\frac{11}{7}$sec. Calculate the velocity and the kinetic energy of the particle at the moment when its displacement is 0.6 metre.

Ans. : We know that, $v = \omega\sqrt{(a^2 - y^2)}$

Further $\omega = \frac{2\pi}{T}$

$\therefore v = \frac{2\pi}{T}\sqrt{(a^2 - y^2)}$

$= \frac{2 \times 30.14}{\left(\frac{11}{7}\right)}\sqrt{[(1.0)^2 - (0.6)^2]}$

$= 3.2m/\sec$

Kinetic energy at displacement is given by

$K = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.8 \times (3.2)^2$

$= 4.1$ joule.

102. Given the example of the motion in the following cases:

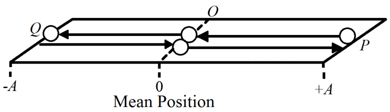
i. Where magnitude and direction of the acceleration of the particle changes.
ii. Where the magnitude and direction of acceleration of body remains constant.
iii. Where magnitude of acceleration changes but its direction remains constants.
iv. Where the magnitude of acceleration remains constant but its direction changes.

Ans. :

i. In S.H.M., acceleration is always proportional to displacement but directed opposite to the displacement. So in this case, magnitude as well as direction of acceleration changes.
ii. A body falling under gravity near the surface of the earth.
iii. A body falling under gravity from a height comparable to the radius of the earth.
iv. A body revolving in a circular path with constant speed.

103. What is the ratio between the distance travelled by the oscillator in one time period and amplitude?

Ans. : In the diagram shown a particle is executing SHM between P and Q. The particle starts from mean position 'O' moves to amplitude position 'P', then particle turn back and moves from 'P' to iQ Finally the particle turns back again and return to mean position 'O'. In this way the particle completes one oscillation in one time period.



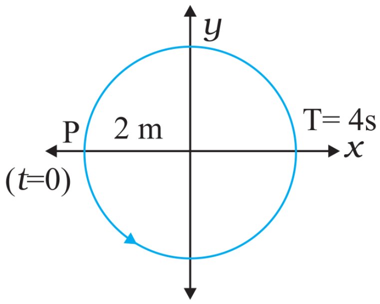
Total distance travelled while it goes from O → P → O → Q → O

= OP + PO + OQ + QO = A + A + A + A = 4A

Amplitude = OP = A

Hence, ratio of distance and amplitude = 4A/ A = 4.

104. Figures correspond to two circular motions. The radius of the circle, the period of revolution, the initial position, and the sense of revolution (i.e. clockwise or anti-clockwise) are indicated on each figure. Obtain the corresponding simple harmonic motions of the x-projection of the radius vector of the revolving particle P, in each case.



Ans. : Let B be any point on the circle of reference of the figure. From B draw BN perpendicular on x-axis.

Then $\triangle \mathrm{BON} = \theta = \omega \mathrm{t}$

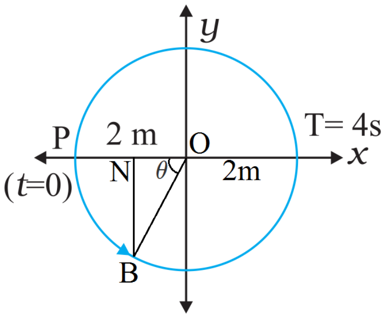
$\therefore$ In $\triangle \mathrm{ONB}, \quad \cos\theta = \frac{\mathrm{ON}}{\mathrm{OB}}$

Or $\mathrm{ON} = \mathrm{OB}\cos\theta$

$\therefore \quad -\mathrm{x} = 2\cos\omega\mathrm{t}$

$\Rightarrow\ x = -2\frac{\cos(2\pi)}{T}t = -2\cos\frac{2\pi}{4}t$

$\therefore\ x = -2\cos\frac{\pi}{4}t$ which is equation of SHM



105. A simple pendulum with a brass has a time period T. The bob is now immersed in a non-viscous liquid and oscillated. If the density of the liquid is $\frac{1}{9}$ that of brass, find the time period of the same pendulum.

Ans. : Let V be the volume and P be the density of the brass bob. Mass of the bob $m = V\rho$ and weight of bob $V\rho g$.

Buoyancy force of liquid on bob $= V\frac{\rho}{9}g$

$= \frac{V\rho g}{9}$

So the effective weight of bob in liquid $V\rho g - \frac{V\rho g}{9}$

$= 8\frac{V\rho g}{9}$

$\therefore$ Acceleration $g' = \frac{\frac{8V\rho g}{9}}{m}$

$= \frac{\frac{8V\rho g}{9}}{V\rho}$

$= \frac{8g}{9}$

Time period of the bob $= 2\pi\sqrt{\frac{l}{g}}$

$= 2\pi\sqrt{\frac{l}{\frac{8g}{9}}}$

$= 2\pi\sqrt{\frac{l}{g}} \times \frac{3}{\sqrt{8}}$

$= \frac{3T}{\sqrt{8}}$

106. Two pendulums of lengths 100cm and 110.25cm start oscillating in phase simultaneously. After how many oscillations will they again be in phase together?

Ans. : $T = 2\pi\sqrt{\frac{l}{g}}$

$l_1 = 100cm$

$l_2 = 110.25cm$

For smaller pendulum, $T_1 = 2\pi\sqrt{\frac{100}{g}} \cdots (i)$

For larger pendulum, $T_2 = 2\pi\sqrt{\frac{110.25}{g}} \cdots (ii)$

Let these pendulums oscillate in phase again if larger pendulum completes 'n' oscillations. It means smaller pendulum must complete (n + 1) oscillations.

$\mathrm{nT_2 = (n+1)T_1}$

$\frac{\mathrm{n+1}}{\mathrm{n}}$

$\frac{\mathrm{T_2}}{\mathrm{T_1}} = \sqrt{\frac{110.25}{100}}$

$= 1.05$

$= 1 + \frac{1}{\mathrm{n}}$

$= 1.05$

$= \frac{1}{\mathrm{n}} = 0.05$

$= \frac{5}{100} = \frac{1}{20}$

$\therefore \mathrm{n} = 20$

Hence both pendulums will again oscillate in phase after 20 oscillations of the larger or 21 oscillations of the smaller pendulum.

107. A block whose mass is 1kg is fastened to a spring. The spring has a spring constant of $50Nm^{-1}$. The block is pulled to a distance x = 10cm from the equilibrium position at x = 0 on a frictionless surface from rest at t = 0. Calculate the kinetic, potential and total energies of the block when it is 5cm away from the mean position.

Ans. : Here mass of block m = 1 kg and spring constant k = 50 Nm^{-1}

$\therefore$ Angular frequency of SHM $\omega = \sqrt{\frac{\mathrm{k}}{\mathrm{m}}}$

$= \sqrt{\frac{50}{1}}$

$= 7.07 \text{ rad s}^{-1}$

As at t = 0, the block was x = 0, and then the block was pulled to distance x = 10cm, it shows that amplitude of oscillation A = 10 - 0 = 10cm = 0.1m. Moreover at any instant the instantaneous displacement x(t) = 5cm = 0.005m

$\therefore$ Kinetic energy of the block $\frac{1}{2}\mathrm{m}\omega^2(\mathrm{A}^2 - \mathrm{x}^2)$

$\frac{1}{2} \times 1 \times (7.07)^2[(0.05)]$

$= 0.19\mathrm{J}$

Potential energy of the block $\mathrm{U} = \frac{1}{3}\mathrm{m}\omega^2\mathrm{x}^2$

$= \frac{1}{2} \times 1 \times (7.07)^2 \times (0.05)^2$

$= 6.25 \times 10^{-2}\mathrm{J}$

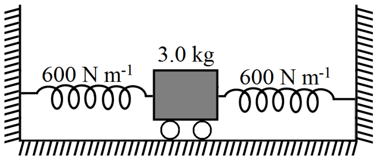
Total energy of thr block $\mathrm{E = K + U}$

$= 0.19\mathrm{J} + 6.25 \times 10^{-2}\mathrm{J}$

$= 0.25\mathrm{J}$

108. A trolley of mass 3.0kg, as shown in Figure, is connected to two springs, each of spring constant $600Nm^{-1}$. If the trolley is displaced from its equilibrium position by 5.0cm and released, what is (a) the period of ensuing oscillations, and (b) the maximum speed of the trolley? How much energy is dissipated as heat by the time trolley comes to rest due

damping forces?



Ans. : Equivalent spring conslant = k' = 2k = 1200Nm^{-1}, m = 3kg

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{3}{1200}}$$

$$= \frac{2\pi}{20} = \frac{\pi}{10}\text{sec.}$$

Maximum speed

$$v = \omega A = 20 \times 5 \times -10^{-2} = 1\text{ms}^{-1}$$

Energy dissipated = Maximum energy

$$= \frac{1}{2}m\omega^2A^2$$

$$= \frac{1}{2} \times 3 \times 20^2 \times 25 \times 10^{-4}$$

$$= 600 \times 25 \times 10^{-4}$$

$$= 150 \times 10^{-2} \text{ Joule}$$

$$= 1.5 \text{ Joule}$$

109. What is the frequency of a second pendulum in an elevator moving up with an acceleration of $\frac{g}{2}$?

Ans. : For second pendulum, frequency $v = \frac{1}{2}s^{-1}$ When elevator is moving upwards with acceleration a, the effective acceleration due to gravity is $g_1 = g + a = g + \frac{g}{2} = \frac{3g}{2}$

Since $v = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$

Hence, $v^2 \propto g$

$$\therefore \frac{v_1}{v_2} = \frac{g_1}{g}$$

$$\frac{\frac{3g}{2}}{g} = \frac{3}{2}$$

$$= \frac{v_1}{v} = \sqrt{\frac{3}{2}}$$

$$= 1.225$$

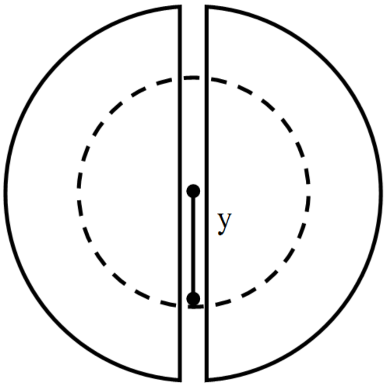
$$\Rightarrow v_1 = 1.225$$

$$= v = 1.225 \times \frac{1}{2}$$

$$= 0.612s^{-1}$$

110. Suppose a tunnel is dug through the earth from one side to the other side along a diameter. Show that the motion of a particle dropped into the tunnel is simple harmonic motion. Find the time period. Neglect all the frictional forces and assume that the earth has a uniform density. G = 6.67 × 10^{-11}Nm^2 kg^{-2}; density of earth = 5.51 × 10^{3}kg m^{-3}

Ans. : Figure shows a tunnel dug along the diameter of the earth. Consider the case of a particle of mass m at a distance y from the centre of the earth. There will be a gravitational attraction of the earth on this particle due to the portion of matter contained in a sphere of radius y. The mass of the sphere of radius y is given by



M = Volume × density

$M = \frac{4}{3}\pi y^3 \times d$

This mass can be regarded as concentrated at the centre of the earth. The force F between this mass and the particle of mass m is given by

$F = -\frac{GMm}{y^2}$

Negative sign shows that the force is of attraction.

$\therefore F = -G\left(\frac{4}{3}\pi y^3 d\right)\frac{m}{y}^2$

$-G\left(\frac{4}{3}\pi md\right)y$

$F \propto y$

The force is directly proportional to the displacement, hence the motion is simple harmonic motion.

Here, the constant $k\frac{4}{3}\pi mdG$.

The time period, $T = 2\pi\sqrt{\frac{m}{k}}$

$= 2\pi\sqrt{\left(\frac{3m}{4\pi mdG}\right)}$

$= 2\pi\sqrt{\left(\frac{3m}{4\pi dG}\right)}$

$= \sqrt{\left(\frac{3\pi}{dG}\right)}$

$= \sqrt{\left(\frac{3\times 3.14}{5.51\times 10^3\times 6.67\times 10^{-11}}\right)}$

$= 42.2 \text{ minutes.}$

111. The mass 'M' attached to a spring oscillates with a period 2s. If the mass is increased by 2kg, the period increases by 1s. Find the initial mass 'M', assuming that Hooke's law is obeyed.

Ans. : Let the initial mass and time periods be M and T respectively. If Hooke's law is obeyed, then the oscillations of the spring will be simple harmonic having time period T given by $T = 2\pi\sqrt{\frac{M}{k}}$ Given T = 2s

$\therefore 2\pi = 2\pi\sqrt{\frac{M}{k}}$ k = spring constant ...(i)

On increasing the mass by 2kg

$3=2\pi\sqrt{\frac{M+2}{k}}\cdots(ii)$

Squaring and dividing equation (ii) by (i) we have

$\frac{9}{4}=\frac{M+2}{M}=1+\frac{2}{M}$

$=\frac{2}{M}=\frac{9}{4}-1=\frac{5}{4}$

$\therefore M=\frac{2\times4}{5}$

$=\frac{8}{5}=1.6kg$

112. An object of mass 0.2kg executes simple harmonic oscillations along the x-axis with a frequency of $\frac{25}{\pi}$ Hertz. At the position x = 0.04m, the object has kinetic energy of 0.5J and potential energy of 0.4J. Find the amplitude of oscillations.

Ans. : Given that, $m=0.2kg, v=\frac{25}{\pi}Hz$

$\therefore \omega=2\pi v=2\pi\times\frac{25}{\pi}$

$=50\text{ rad s}^{-1}$

$x=0.04m, E_K=0.5J,$

$E_p=0.4J$

We know,

$E_K=\frac{1}{2}m\omega^2(a^2-x^2)$

and $E_P=\frac{1}{2}m\omega^2x^2$

Dividing the two,

$\frac{E_K}{E_P}=\frac{a^2-x^2}{x^2}=\frac{a^2}{x^2}-1$

$\frac{a^2}{x^2}=\frac{E_K}{E_P}+1$

$a^2=\left(\frac{E_K}{E_P}+1\right)x^2$

$=\left(\frac{0.5}{0.4}+1\right)(0.04)^2$

$=\frac{9}{4}\times0.04\times0.04$

$=36\times10^{-4}m^2$

$a=\sqrt{36\times10^{-4}}m=6\times10^{-2}m$

113. A spring compressed by 0.1m develops a restoring force 10N. A body of mass 4kg placed on it. Deduce

i. The force constant of the spring.

ii. The depression of the spring under the weight of the body(take g = 10 N/ kg).

iii. The period of oscillation, the body is distributed and.

iv. Frequency of oscillation.

Ans. : Here,$F=10N$

$\Delta l=0.1m$

$m=4kg$

i. $k=\frac{F}{\Delta l}$

$= \frac{10}{0.1} = 100\text{Nm}^{-1}$

ii. $y = \frac{mg}{k}$

$= \frac{4\times10}{100} = 0.4\text{m}$

iii. $T = 2\pi\sqrt{\frac{m}{k}}$

$= 2 \times \frac{22}{7}\sqrt{\frac{4}{100}}$

$= 1.26\text{s}$

iv. Frequency. $v = \frac{1}{T}$

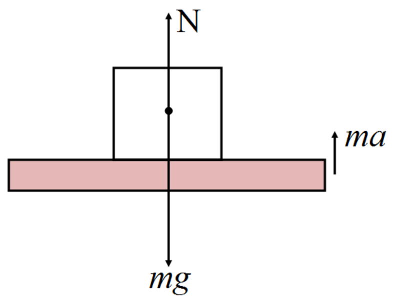
$= \frac{1}{1.26} = 0.8\text{Hz}$

114. A person normally weighing 50kg stands on a massless platform which oscillates up and down harmonically at a frequency of $2.0s^{-1}$ and an amplitude 5.0cm. A weighing machine on the platform gives the persons weight against time.

a. Will there be any change in weight of the body, during the oscillation?

b. If answer to part (a) is yes, what will be the maximum and minimum reading in the machine and at which position?

Ans. : Weight in weight machine will be due to the normal reaction (N) by platform. Consider the top position of platform, two
forces due to weight of person and oscillator acts both downward.



So motion is downward. Let with acceleration a then

ma = mg - N ...(i)

When platform lifts form its lowest position to upward

ma = N - mg ...(ii)

$a\omega^2 A$ acceleration of oscillator

i. Form (i) equation

$N = mg - m\omega^2 A$

Where A is amplitude, ω angular frequency, m mass of oscillator.

$\omega = 2\pi v = 2\pi \times 2 = 4\pi \text{ rad/sec.}$

$A = 5\text{cm} = 5 \times 10^{-2}\text{m}_2\text{m} = 50\text{kg}$

$N = 50 \times 9.8 - 50 \times 4\pi \times 4\pi \times 5 \times 10^{-2}$

$= 50[9.8 - 16\pi^2 \times 5 \times 10^{-2}$

$= 50[9.8 - 80 \times 3.14 \times 3.14 \times 10^{-2}]$

$N = 50[9.8 - 7.89] = 50 \times 1.91 = 95.50\text{N}$

So minimum weight is 95.50N.

ii. form (ii) N – mg = ma

For upward motion form lowest point of oscillator.

$$N = mg + ma = m(a+g)$$
$$= m[9.81 + \omega^2 A] a = \omega^2 A$$
$$= 50[9.81 + 16\pi^2 \times 5 \times 10^{-2}$$
$$= 50[9.81 + 7.89] = 50[17.70]$$
$$N = 885.00N$$

a. Hence, there is a change in weight of the body during oscillation.

b. The maximum weight is 885N, when platform moves from lowest to up direction.

And the minimum is 95.5N, when platform moves from highest point to downward direction.

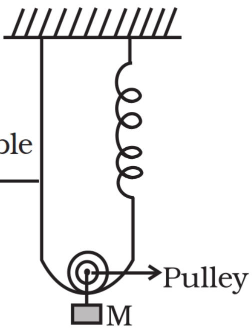
115. Find the time period of mass M when displaced from its equilibrium positon and then released for the system shown in figure.

Ans. : Key concept: For observing oscillation, we have to displace the block slightly beyond equilibrium position and find the acceleration due to the restoring force.

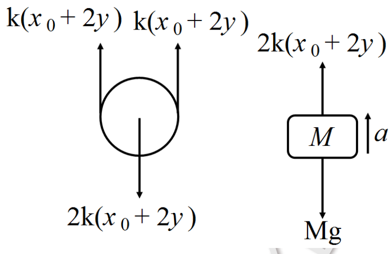
Let in the equilibrium position, the spring has extended by an amount x_0.

Tension in the spring = kx_0

For equilibrium of the mass M, Mg = $2kx_0$



Let the mass be pulled through a distance y and then released. But, string is inextensible, hence the spring alone will contribute the total extension y + y = 2y, to lower the mass down by y from initial equilibrium mean position x_0. So, net extension in the spring (x_0 + 2y). From F.B.D of the block,



$$2K(x_0 + 2y) - Mg = Ma$$
$$2Kx_0 + 4ky - Mg = Ma \Rightarrow Ma = 4ky$$
$$\bar{a} = -\left(\frac{4k}{M}\right)\bar{y}$$

k and M being constant.

$\therefore a \propto -x$. Hence, motion is SHM.

Comparing the above acceleration expression with standard SHM equation.

$a = -\omega^2 x$, we get

$\omega^2 = \frac{4k}{M} \Rightarrow \omega = \sqrt{\frac{4k}{M}}$

116. In figure k = 100N/m, M = 1kg and F = 10N,

a. Find the compression of the spring in the equilibrium position.

b. A sharp blow by some external agent imparts a speed of 2m/s to the block towards left. Find the sum of the potential energy of the spring and the kinetic energy of the block at this instant.

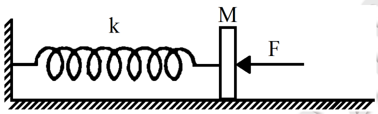
c. Find the time period of the resulting simple harmonic motion.

d. Find the amplitude.

e. Write the potential energy of the spring when the block is at the left extreme.

f. Write the potential energy of the spring when the block is at the right extreme.

The answers of (b), (e) and (f) are different. Explain why this does not violate the principle of conservation of energy.



Ans. : Given, k = 100N/m, M = 1kg and F = 10N

a. In the equilibrium position,

compression $\delta = \frac{F}{k} = \frac{10}{100} = 0.1m = 10cm$

b. The blow imparts a speed of 2m/s to the block towards left.

$\therefore P.E. + KE = \frac{1}{2}k\delta^2 + \frac{1}{2}Mv^2$

$= \left(\frac{1}{2}\right) \times 100 \times (0.1)^2 + \left(\frac{1}{2}\right) \times 1 \times 4$

$= 0.5 + 2 = 2.5J$

c. Time period $= 2\pi\sqrt{\frac{M}{k}} = 2\pi\sqrt{\frac{1}{100}} = \frac{\pi}{5}sec$

d. Let the amplitude be 'x' which means the distance between the mean position and the extreme position.

So, in the extreme position, compression of the spring is $(x + \delta)$.

Since, in SHM, the total energy remains constant.

$= \left(\frac{1}{2}\right)k(x+\delta)^2$

$= \left(\frac{1}{2}\right)k\delta^2 + \left(\frac{1}{2}\right)mv^2 + Fx$

$= 2.5 + 10x$

$\left[\text{because } \left(\frac{1}{2}\right)k\delta^2 + \left(\frac{1}{2}\right)mv^2 = 2.5\right]$

So, $50(x + 0.1)^2 = 2.5 + 10x$

$\therefore 50x^2 + 0.5 + 10x = 2.5 + 10x$

$\therefore 50x^2 = 2$

$\Rightarrow x^2 = \frac{2}{50} = \frac{4}{100}$

$\Rightarrow x = \frac{2}{10}m = 20cm.$

e. Potential Energy at the left extreme is given by,

$P.E. = \left(\frac{1}{2}\right)k(x+\delta)^2$

$= \left(\frac{1}{2}\right) \times 100(0.1 + 0.2)^2$

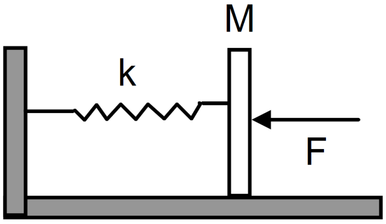
$= 50 \times 0.09 = 4.5\mathrm{J}$

f. Potential Energy at the right extreme is given by,

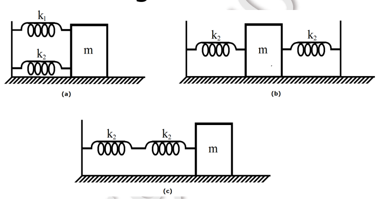
$\mathrm{P.E.} = \left(\frac{1}{2}\right)\mathrm{k}(\mathrm{x}+\delta)^2 - \mathrm{F}(2\mathrm{x})$ [2x = distance between two extremes]

$= 4.5 - 10(0.4) = 0.5\mathrm{J}$

The different values in (b) (e) and (f) do not violate law of conservation of energy as the work is done by the external force 10N.



117. Find the time period of the oscillation of mass m in figure What is the equivalent spring

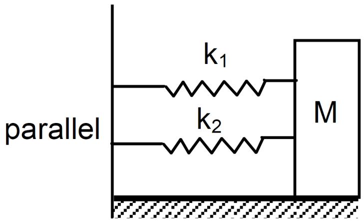


constant of the pair of springs in each case?

Ans. :

a. Equivalent spring constant k = $k_1 + k_2$ (parallel)

$\mathrm{T} = 2\pi\sqrt{\frac{\mathrm{M}}{\mathrm{k}}} = 2\pi\sqrt{\frac{\mathrm{m}}{\mathrm{k}_1+\mathrm{k}_2}}$



b. Let us, displace the block m towards left through displacement ‘x’

Resultant force $\mathrm{F} = \mathrm{F}_1 + \mathrm{F}_2 = (\mathrm{k}_1+\mathrm{k}_2)\mathrm{x}$

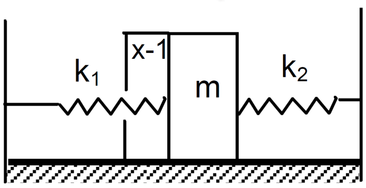
Acceleration $\left(\frac{\mathrm{F}}{\mathrm{m}}\right) = \frac{(\mathrm{k}_1+\mathrm{k}_2)\mathrm{x}}{\mathrm{m}}$

Time period $\mathrm{T} = 2\pi\sqrt{\frac{\text{displacement}}{\text{Acceleration}}}$

$= 2\pi\sqrt{\frac{\mathrm{x}}{\frac{\mathrm{m}(\mathrm{k}_1+\mathrm{k}_2)}{\mathrm{m}}}}$

$= 2\pi\sqrt{\frac{\mathrm{m}}{\mathrm{k}_1+\mathrm{k}_2}}$

The equivalent spring constant k = $k_1 + k_2$

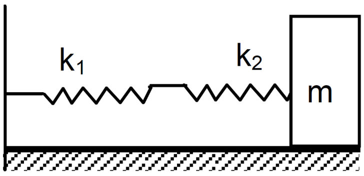


c. In series conn equivalent spring constant be k.

So, $\frac{1}{\mathrm{k}} = \frac{1}{\mathrm{k}_1} + \frac{1}{\mathrm{k}_2} = \frac{\mathrm{k}_2+\mathrm{k}_1}{\mathrm{k}_1\mathrm{k}_2}$

$$\Rightarrow k = \frac{k_1 k_2}{k_1 + k_2}$$

$$T = 2\pi\sqrt{\frac{M}{k}} = 2\pi\sqrt{\frac{m(k_1+k_2)}{k_1 k_2}}$$

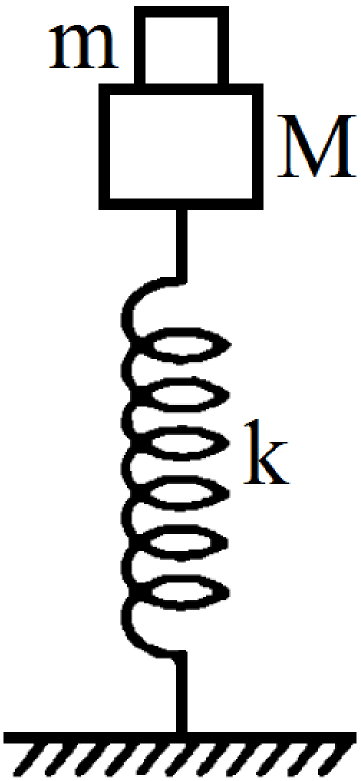


118. A small block of mass m is kept on a bigger block of mass M which is attached to a vertical spring of spring constant k as shown in the figure. The system oscillates vertically.

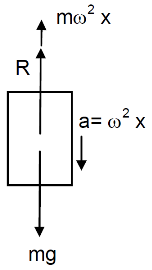
a. Find the resultant force on the smaller block when it is displaced through a distance x above its equilibrium position.

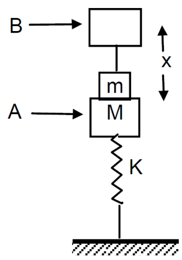
b. Find the normal force on the smaller block at this position. When is this force smallest in magnitude?

c. What can be the maximum amplitude with which the two blocks may oscillate together?



Ans. :





a. From the free body diagram,

$$\therefore R + m\omega^2 x - mg = 0 \ldots (1)$$

Resultant force $m\omega^2 x = mg - R$

$\Rightarrow m\omega^2 x = m\left(\frac{k}{M+m}\right)$

$\Rightarrow x = \frac{mkx}{M+m}$

$\left[\omega = \sqrt{\frac{k}{M+m}} \text{ for spring mass system}\right]$

b. $R = mg - m\omega^2 x = mg - m\frac{k}{M+m}x = mg - \frac{mkx}{M+m}$

For R to be smallest, $m\omega^2 x$ should be max. i.e. x is maximum.

The particle should be at the high point.

c. We have $R = mg - m\omega^2 x$

The tow blocks may oscillates together in such a way that R is greater than 0. At limiting condition, $R = 0,\ mg = m\omega^2 x$

$x = \frac{mg}{m\omega^2} = \frac{mg(M+m)}{mk}$

So, the maximum amplitude is $= \frac{g(M+m)}{k}$

119. A particle is subjected to two simple harmonic motions given by $x_1 = 2.0\sin(100\pi t)$ and $x_2 = 2.0\sin\left(120\pi t + \frac{\pi}{3}\right)$ where x is in centimeter and t in second. Find the displacement of the particle at

a. t = 0.0125.

b. t = 0.025.

Ans. : $x_1 = 2\sin 100\pi t$

$x_2 = 2\sin\left(120\pi t + \frac{\pi}{3}\right)$

So, resultant displacement is given by,

$x = x_1 + x_2 = 2\left[\sin(100\pi t) + \sin\left(120\pi t + \frac{\pi}{3}\right)\right]$

a. At $t = 0.0125s,$

$x = 2\left[\sin(100\pi \times 0.0125) + \sin\left(120\pi \times 0.0125 + \frac{\pi}{3}\right)\right]$

$= 2\left[\sin\frac{5\pi}{4} + \sin\left(\frac{3\pi}{2} + \frac{\pi}{3}\right)\right]$

$= 2[(-0.707) + (-0.5)] = -2.41cm$

b. At $t = 0.025s.$

$x = 2\left[\sin(100\pi \times 0.025) + \sin\left(120\pi \times 0.025 + \frac{\pi}{3}\right)\right]$

$= 2\left[\sin\frac{5\pi}{2} + \sin\left(3\pi + \frac{\pi}{3}\right)\right]$

$= 2[1 + (-0.8666)] = 0.27cm.$

120. A particle having mass 10g oscillates according to the equation $x = (2.0cm)\sin\left[(100s^{-1})t + \frac{\pi}{6}\right]$. Find (a) the amplitude, the time period and the spring constant (b) the position, the velocity and the acceleration at t = 0.

Ans. : $x = (2.0cm)\sin\left[(100s^{-1})t + \left(\frac{\pi}{6}\right)\right]$

m = 10g.

a. Amplitude = 2cm

$\omega = 100\,\text{sec}^{-1}$

$\therefore T = \frac{2\pi}{100} = \frac{\pi}{50}\text{sec} = 0.063\,\text{sec.}$

We know that $T = 2\pi\sqrt{\frac{m}{k}}$

$\Rightarrow T^2 = 4\pi^2 \times \frac{m}{k}$

$\Rightarrow k = \frac{4\pi^2}{T^2}m$

$= 10^5\text{dyne/cm} = 100\text{N/m}\left[\text{because } \omega = \frac{2\pi}{T} = 100\,\text{sec}^{-1}\right]$

b. At t = 0

$x = 2\text{cm}\sin\left(\frac{\pi}{6}\right) = 2 \times \left(\frac{1}{2}\right) = 1\text{cm.}$ from the mean position.

We know that $x = A\sin(\omega t + \phi)$

$v = A\cos(\omega t + \phi)$

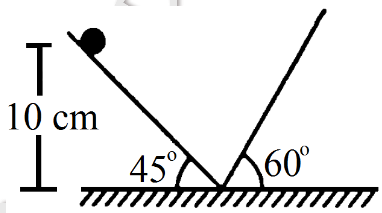
$= 2 \times 100\cos\left(0 + \frac{\pi}{6}\right) = 200 \times \frac{\sqrt{3}}{2}$

$= 100\sqrt{3}\,\text{sec}^{-1} = 1.73\text{m/s}$

c. $a = -\omega^2 x = 100^2 \times 1$

$= 100\text{m/s}^2$

121. Find the time period of the motion of the particle shown in figure Neglect the small



effect of the bend near the bottom.

Ans. : Let the time taken to travel AB and BC be t_1 and t_2 respectively

Fro part AB, $a_1 = g\sin 45°$.

$s_1 = \frac{0.1}{\sin 45°} = 2\text{m}$

Let, v = velocity at B

$\therefore v^2 - u^2 = 2a_1 s_1$

$\Rightarrow v^2 = 2 \times g\sin 45° \times \frac{0.1}{\sin 45°} = 2$

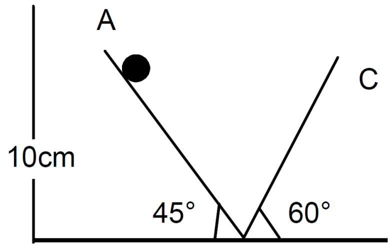
$\Rightarrow v\sqrt{2}\text{m/s}$

$\therefore t_1 = \frac{v-u}{a_1} = \frac{\sqrt{2}-0}{\frac{g}{\sqrt{2}}} = \frac{2}{g} = \frac{2}{10} = 0.2\,\text{sec}$

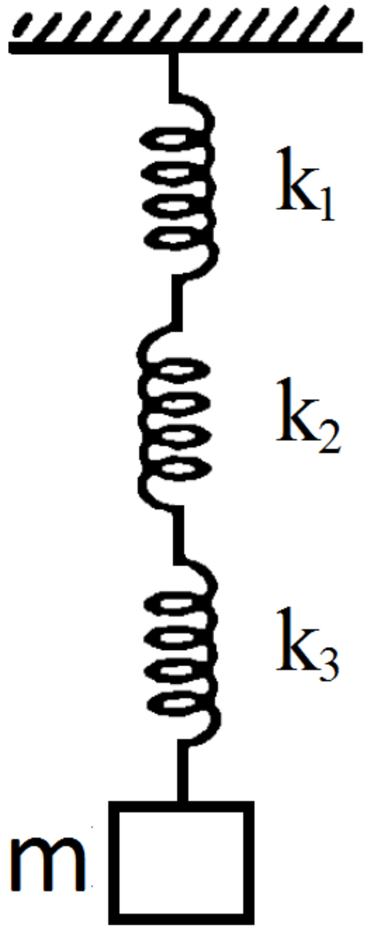
Again for part BC, $a_2 = -g\sin 60°,\; u = \sqrt{2}, v = 0$

$\therefore t_2 = \frac{0-\sqrt{2}}{-g\left(\frac{\sqrt{3}}{2}\right)} = \frac{2\sqrt{2}}{\sqrt{3}g} = \frac{2\times(1.414)}{(1.732)\times 10} = 0.165\,\text{sec.}$

So, time period $= 2(t_1 + t_2) = 2(0.2 + 0.155) = 0.71\,\text{sec}$



122. Find the elastic potential energy stored in each spring shown in figure when the block is in equilibrium. Also find the time period of vertical oscillation of the block.



Ans. : k_1, k_2, k_3 are in series,

$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$

$\Rightarrow k = \frac{k_1 k_2 k_3}{k_1 k_2 + k_2 k_3 + k_1 k_3}$

Time period $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m(k_1 k_2 + k_2 k_3 + k_1 k_3)}{k_1 k_2 k_3}}$

$= 2\pi\sqrt{m\left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}\right)}$

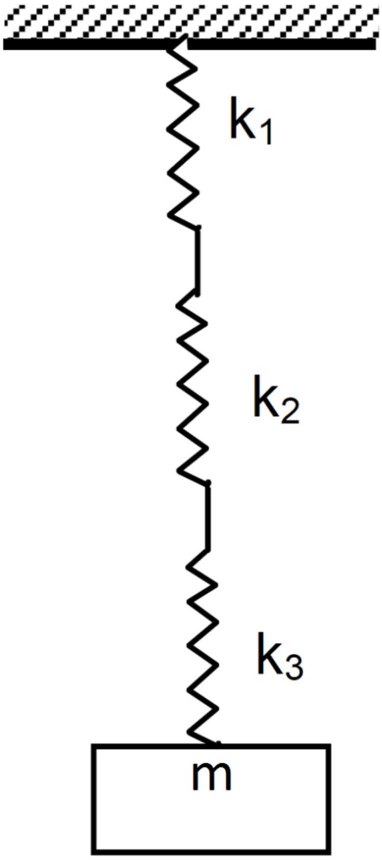
Now, Force = weight = mg.

$\therefore$ At k_1 spring, $x_1 = \frac{mg}{k_1}$

Similarly $x_2 = \frac{mg}{k_2}$ and $x_3 = \frac{mg}{k_3}$

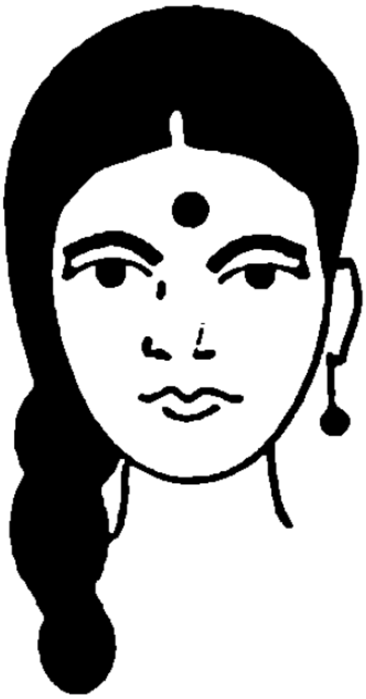
$\therefore PE_1 = \left(\frac{1}{2}\right)k_1 x_1^2 = \frac{1}{2}k_1\left(\frac{mg}{k_1}\right)^2 = \frac{1}{2}k_1\frac{m^2g^2}{k_1^2} = \frac{m^2g^2}{2k_1}$

Similarly $\mathrm{PE}_2 = \frac{\mathrm{m}^2\mathrm{g}^2}{2\mathrm{k}_2}$ and $\mathrm{PE}_3 = \frac{\mathrm{m}^2\mathrm{g}^2}{2\mathrm{k}_3}$

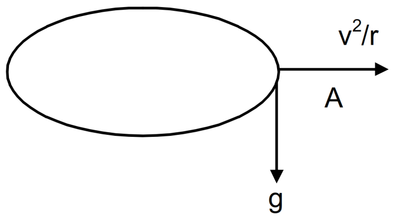


*** Case study based questions** **[16]**

123. The ear-ring of a lady shown in has a 3cm long light suspension wire.
   a. Find the time period of small oscillations if the lady is standing on the ground.
   b. The lady now sits in a merry-go-round moving at 4m/s in a circle of radius 2m. Find the time period of small oscillations of the ear-ring.



Ans. :



a. $\ell = 3\text{cm} = 0.03\text{m}.$

$T = 2\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{0.03}{9.8}} = 0.34$ secound.

b. When the lady sets on the Merry-go-round the ear rings also experience centrepetal acceleration,

$a = \frac{v^2}{r} = \frac{4^2}{2} = 8\text{m/s}^2$

Resultant Acceleration $A = \sqrt{g^2 + a^2} = \sqrt{100 + 64} = 12.8\text{m/s}^2$

Time period $T = 2\pi\sqrt{\frac{\ell}{A}} = 2\pi\sqrt{\frac{0.03}{12.8}} = 0.30$ second.

124. A simple pendulum fixed in a car has a time period of 4 seconds when the car is moving uniformly on a horizontal road. When the accelerator is pressed, the time period changes to 3.99 seconds. Making an approximate analysis, find the acceleration of the car.

Ans. : When the car moving with uniform velocity,

$T = 2\pi\sqrt{\frac{\ell}{g}}$

$\Rightarrow 4 = 2\pi\sqrt{\frac{\ell}{g}} \ldots (1)$

When the car makes accelerated motion, let the acceleration be a_0

$T = 2\pi\sqrt{\frac{\ell}{g^2 + a_0^2}}$

$\Rightarrow 3.99 = 2\pi\sqrt{\frac{\ell}{g^2 + a_0^2}}$

Now, $\frac{T}{T'} = \frac{4}{3.99} = \frac{\left(g^2 + a_0^2\right)^{\frac{1}{4}}}{\sqrt{g}}$

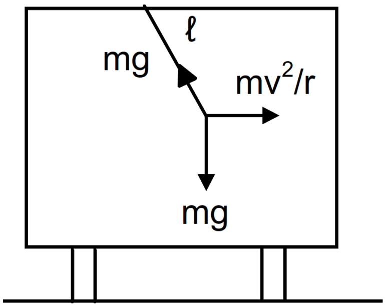
Solving for 'a_0' we can get $a_0 = \frac{g}{10}\text{ms}^{-2}$

125. A simple pendulum of length l is suspended from the ceiling of a car moving with a speed v on a circular horizontal road of radius r.

a. Find the tension in the string when it is at rest with respect to the car.

b. Find the time period of small oscillation.

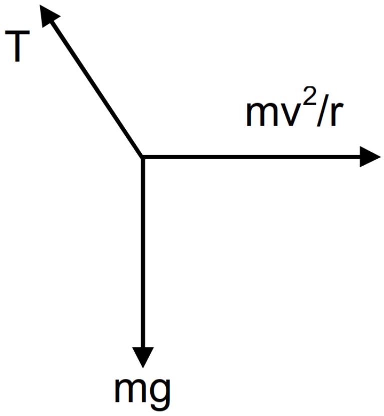
Ans. :



From the freebody diagram,

$$T = \sqrt{(mg)^2 + \left(\frac{mv^2}{r^2}\right)}$$

$= m\sqrt{g^2 + \frac{v^4}{r^2}} = ma$ where a = acceleration $= \left(g^2 + \frac{v^4}{r^2}\right)^{\frac{1}{2}}$



The time period of small accellations is given by,

$$T = 2\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{\ell}{\left(g^2+\frac{v^4}{r^2}\right)^{\frac{1}{2}}}}$$

126. A person goes to bed at sharp 10:00 pm every day. Is it an example of periodic motion? If yes, what is the time period? If no, why?

Ans. : It is not motion at first place.

----- A person who never made a mistake never tried anything new. - (Albert Einstein) -----